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ORIGINAL ARTICLE

Steady nanofluid flow between parallel plates considering thermophoresis and Brownian effects

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Abstract In this article, heat and mass transfer behavior of steady nanofluid flow between parallel plates in the presence of uniform magnetic field is studied. The important effect of Brownian motion and thermophoresis has been included in the model of nanofluid. The governing equations are solved via the Differential Transformation Method. The validity of this method was verified by comparison of previous work which is done for viscous fluid. The analysis is carried out for different parameters namely: viscosity parameter, Magnetic parameter, thermophoretic parameter and Brownian parameter. Results reveal that skin friction coefficient enhances with rise of viscosity and Magnetic parameters. Also it can be found that Nusselt number augments with an increase of viscosity parameters but it decreases with augment of Magnetic parameter, thermophoretic parameter and Brownian parameter.

1. Introduction

Because of increment in energy price, heat transfer (HT) management is important in energy systems. Recently, nanofluid technology is planned and studied by some researchers experimentally or numerically in order to enhance HT process. The nanofluid can be applied to engineering problems, such as heat exchangers, cooling of electronic equipment and chemical processes. In most of the studies, it is assumed that nanofluid treats as the common pure fluid. Abu-Nada et al. (2008)

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Nomenclature

C	nanofluid concentration	*Greek symbols*	
C_f	skin friction coefficients	α	thermal diffusivity
C_p	specific heat at constant pressure	ϕ	dimensionless concentration
h	distance between the plates	η	dimensionless variable
k	thermal conductivity	μ	dynamic viscosity
Nu	Nusselt number	υ	kinematic viscosity
p^*	modified fluid pressure	θ	dimensionless temperature
Pr	Prandtl number	ρ	fluid density
R	viscosity parameter	σ	electrical conductivity
u, v, w	velocity components along x, y, z axes respectively	τ_w	skin friction or shear stress along the stretching surface
$u_w(x)$	velocity of the stretching surface		

investigated the enhancement of natural convection in horizontal concentric annuli field. They concluded that as Rayleigh number decreases, the thermal conductivity effects of nanoparticles cause more HT enhancement. Jou and Tzeng (2006) numerically studied the natural convection enhancements of nanofluid within a two-dimensional enclosure. They analyzed HT performance using Khana-fer's model for various parameters, such as volume fraction, aspect ratio of the enclosure, and Grashof number. Results showed that increasing the buoyancy parameter and volume fraction of nanofluid causes an increase in the average HT coefficient. Rashidi et al. (2013) modeled the application of the second law of thermodynamics to an electrically conducting incompressible nanofluid fluid flowing over a porous rotating disk.

Malvandi and Ganji (2014a) studied the laminar flow and convective HT of alumina/water nanofluid inside a circular microchannel in the presence of a uniform magnetic field. For smaller nanoparticles, more uniform volume fraction is observed and abnormal variations in the HT rate are vanished. MHD effect on natural convection HT in an inclined L-shape enclosure filled with nanofluid was studied by Sheikholeslami et al. (2014). They found that enhancement in HT has reverse relationship with Hartmann number and Rayleigh number. They concluded that using magnetic rotating disk affects HT rate in renewable energy systems and industrial thermal management. Recently several authors investigated nanofluid flow and HT enhancement applications (Cortell, 2014; Mabood et al., 2014; Garoosi et al., 2015a,b,c; Ashorynejad et al., 2013a,b Hatami and Ganji, 2014a,b; Hatami et al., 2014a–c; Domairry et al., 2012; Sheikholeslami et al., 2013a; Sheikholeslami and Ganji, 2013; Sheikholeslami et al., 2013b–e; Kefayati, 2013a,b; Kefayati, 2013c; Sheikholeslami et al., 2012a–c; Shehzad et al., 2012; Shehzad et al., 2013a,b; Shehzad et al., 2014a,b; Sheikholeslami et al., 2014a; Sheikholeslami et al., 2014b–f; Sheikholeslami Kandelousi, 2014a,b; Sheikholeslami and Ganji, 2015a; Sheikholeslami et al., 2015a–c; Sheikholeslami and Rashidi, 2015; Ellahi, 2013; Ellahi et al., 2012, 2013, 2015; Raptis, 1998; Rashidi et al., 2015; Akbar et al., 2014a,b; Umavathi and Mohite, 2014: Zeeshan et al., 2014). In all the above articles, it is assumed that there are no slip velocities between nanoparticles and fluid molecules and assumed that the nanoparticle concentration is uniform. Nield and Kuznetsov (2009) studied the natural convection in a horizontal layer of a porous medium saturated by a nanofluid. Khan and Pop (2010) investigated boundary-layer flow of a nanofluid past a stretching sheet as a first paper in that field. Their model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. They have taken into account the Prandtl number, Lewis number, Thermophores number, and Brownian motion number. The investigation of heat and mass transfer unsteady squeezing viscous flow between two parallel plates in motion normal to their own surfaces independent of each other and arbitrary with respect to time has been regarded as one of the most important research topics due to its wide spectrum of scientific and engineering applications such as hydrodynamical equipment, lubrication system, polymer processing, chemical processing tool, materials damage due to freezing, food processing and cooling towers. Mahmood et al. (2007) investigated the HT characteristics in the squeezed flow over a porous surface. Differential Transformation Method is one of the semi-exact methods which do not have the limitation of the perturbation method. Against the traditional higher-order Taylor series procedure, this method applies a polynomial solution that is computationally expensive for higher orders. DTM is a substitute method and its main advantage is applying the nonlinear differential equations without discretization and linearization. This method was introduced by Zhou (1986), by applying the DTM method to different problems in electrical applications. Many researchers investigated Multi-step DTM in different applications, for example Hatami and his colleagues (2014d) studied spherical particles motion in plane Couette fluid flow (Yang and Baleanu, 2013; Cattani et al., 2015 El-Zahar, 2013; Sheikholeslami and Ganji, 2015b: Rashidi, 2009).

The main purpose of this study is to investigate the problem of unsteady nanofluid flow between parallel plates using the Differential Transformation Method. The influence of the radiation parameter, squeeze number, Hartmann number, Brownian motion parameter and thermophoretic parameter on temperature and concentration profiles is investigated.

2. Governing equations

Consider the steady nanofluid flow between two horizontal parallel plates when the fluid and the plates rotate together around the y-axis which is normal to the plates with an angular velocity. A Cartesian coordinate system is considered as

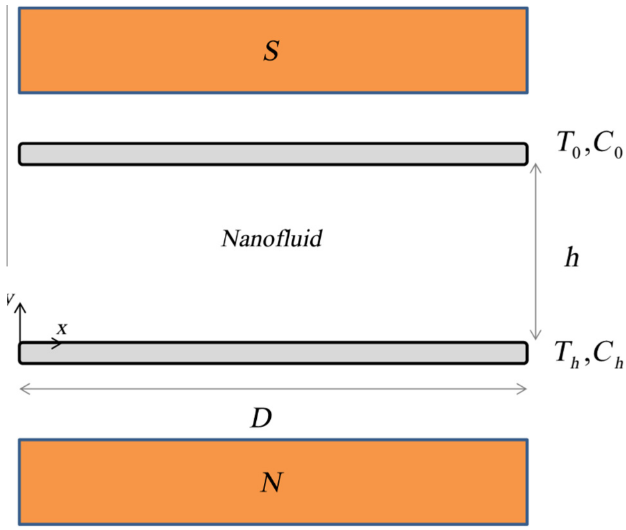


Fig. 1 Geometry of problem.

Table 1 Some4 of the basic operations of Differential Transformation Method.

Original function	Transformed function
$f(\eta) = \alpha g(\eta) \pm \beta h(\eta)$	$F[k] = \alpha G[k] \pm \beta H[k]$
$f(\eta) = \frac{d^n g(\eta)}{d\eta^n}$	$F[k] = \frac{(k+n)!}{k!} G[k+n]$
$f(\eta) = g(\eta)h(\eta)$	$F[k] = \sum_{m-0}^{k} F[m]H[k-m]$
$f(\tau) = \sin(\varpi\eta + \alpha)$	$F[k] = \frac{\varpi^k}{k!}\sin(\frac{\pi k}{2} + \alpha)$
$f(\tau) = \cos(\varpi\eta + \alpha)$	$F[k] = \frac{\varpi^k}{k!}\cos(\frac{\pi k}{2} + \alpha)$
$f(\eta) = e^{\lambda\eta}$	$F[k] = \frac{\lambda^k}{k!}$
$F(\eta) = (1+\eta)^m$	$F[k] = \frac{m(m-1)\ldots(m-k+1)}{k!}$
$f(\eta) = \eta^m$	$F[k] = \delta(k-m) = \begin{cases} l1, & k = m \\ 0, & k \neq m \end{cases}$

followed: the x-axis is along the plate, the y-axis is perpendicular to it (see Fig. 1). The plates are located at $y = 0$ and $y = h$. The lower plate is being stretched by two equal and opposite forces so that the position of the point (0,0,0) remains unchanged. A uniform magnetic flux with density B_0 is acting along y-axis about which the system is rotating. The governing equations in a rotating frame of reference are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho_f\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p^*}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \sigma B_0^2 u, \tag{2}$$

$$\rho_f\left(u\frac{\partial v}{\partial y}\right) = -\frac{\partial p^*}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right), \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{(\rho c_P)_p}{(\rho c_P)_f} \times \left[D_B\left\{\frac{\partial C}{\partial x}\cdot\frac{\partial T}{\partial x} + \frac{\partial C}{\partial y}\cdot\frac{\partial T}{\partial y}\right\} + (D_T/T_c)\left\{\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right\}\right], \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + \left(\frac{D_T}{T_0}\right)\left\{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right\} \tag{5}$$

Here u and v are the velocities in the x and y directions respectively. And T, C, ρ_f, μ, k, c_P, and D_B are temperature, concentration, base fluid's density, dynamic viscosity, thermal conductivity, specific heat of nanofluid, and diffusion coefficient of the diffusing species. Also p^* is the modified fluid pressure. The relevant boundary conditions are:

$$\begin{array}{ll} u = ax,\ v = 0,\ T = T_h,\ C = C_h & at \quad y = 0 \\ u = 0, v = 0,\ T = T_0,\ C = C_0 & at \quad y = +h \end{array} \tag{6}$$

The following non-dimensional variables are introduced:

$$\begin{array}{l} \eta = \frac{y}{h}, \quad u = axf'(\eta), \quad v = -ahf(\eta), \\ \theta(\eta) = \frac{T - T_h}{T_0 - T_h}, \quad \phi(\eta) = \frac{C - C_h}{C_0 - C_h} \end{array} \tag{7}$$

Therefore, the governing equations and boundary conditions for this case in non-dimensional form are given by:

$$f^{iv} - R(f'f'' - ff'') - Mf'' = 0 \tag{8}$$

$$\theta'' + \Pr Rf\theta' + Nb\,\phi'\theta' + Nt\,\theta'^2 = 0, \tag{9}$$

$$\phi'' + R.Scf\phi' + \frac{Nt}{Nb}\theta'' = 0, \tag{10}$$

With these boundary conditions:

$$\begin{array}{ll} f = 0,\ f' = 1,\ \theta = 1,\ \phi = 1 & at \quad \eta = 0 \\ f = 0,\ f' = 0,\ \theta = 0,\ \phi = 0 & at \quad \eta = 1 \end{array} \tag{11}$$

The non-dimensional quantities are defined through in which R is the viscosity parameter, M is the Magnetic parameter, Pr is the Prandtl number, Sc is the Schmidt number, Nb is the Brownian motion parameter and Nt is the thermophoretic parameter.

$$\begin{array}{l} R = \frac{ah^2}{\upsilon}, \quad M = \frac{\sigma B_0^2 h^2}{\rho\upsilon}, \\ \Pr = \frac{\mu}{\rho_f\alpha}, \quad Sc = \frac{\mu}{\rho_f D}, \\ Nb = (\rho c)_p D_B(C_h)/((\rho c)_f\alpha), \\ Nt = (\rho c)_p D_T(T_H)/[(\rho c)_f\alpha T_c]. \end{array} \tag{12}$$

Skin friction coefficient C_f along the stretching wall and Nusselt number Nu along the stretching wall are defined as

$$C_f^* = \left(\frac{Rx}{h}\right)C_f = f''(0), \quad Nu = -\theta' \tag{13}$$

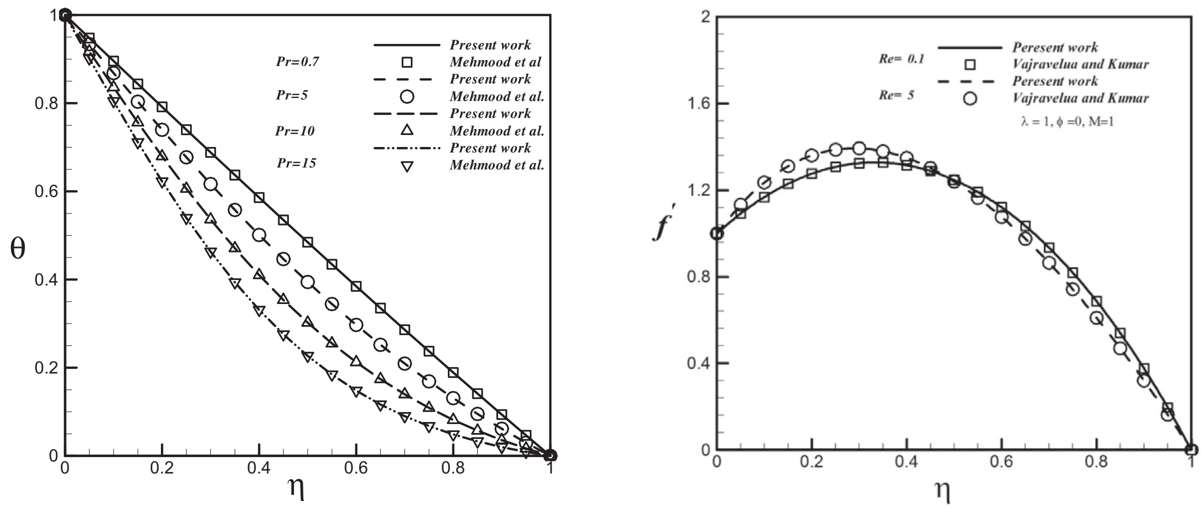
3. Differential Transform Method (DTM)

3.1. Basic of DTM

Basic definitions and operations of differential transformation are introduced as follows. Differential transformation of the function $f(\eta)$ is defined as follows:

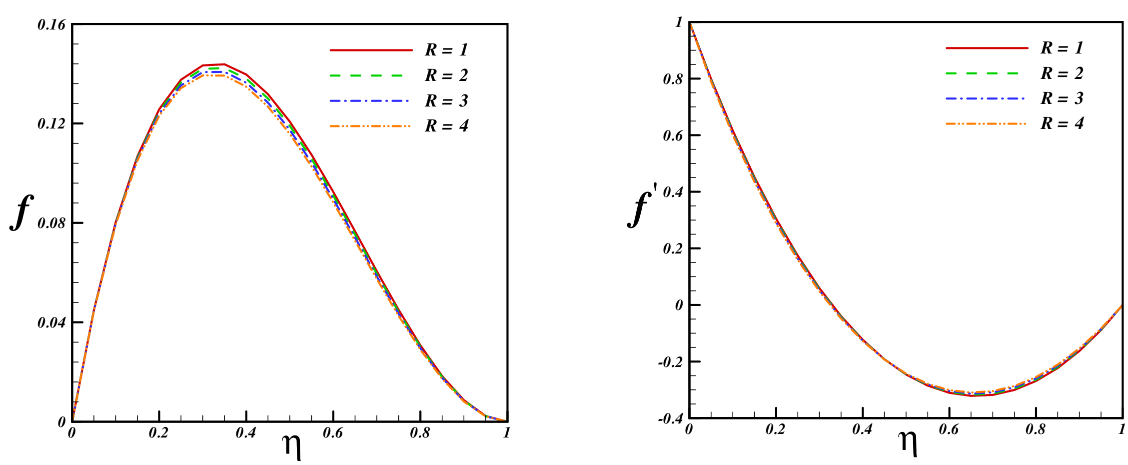
$$F(k) = \frac{1}{k!}\left[\frac{d^k f(\eta)}{d\eta^k}\right]_{\eta=\eta_0} \tag{14}$$

In (14), $f(\eta)$ is the original function and $F(k)$ is the transformed function which is called the T-function (it is also called the spectrum of the $f(\eta)$ at $\eta = \eta_0$, in the k domain). The differential inverse transformation of $F(k)$ is defined as:

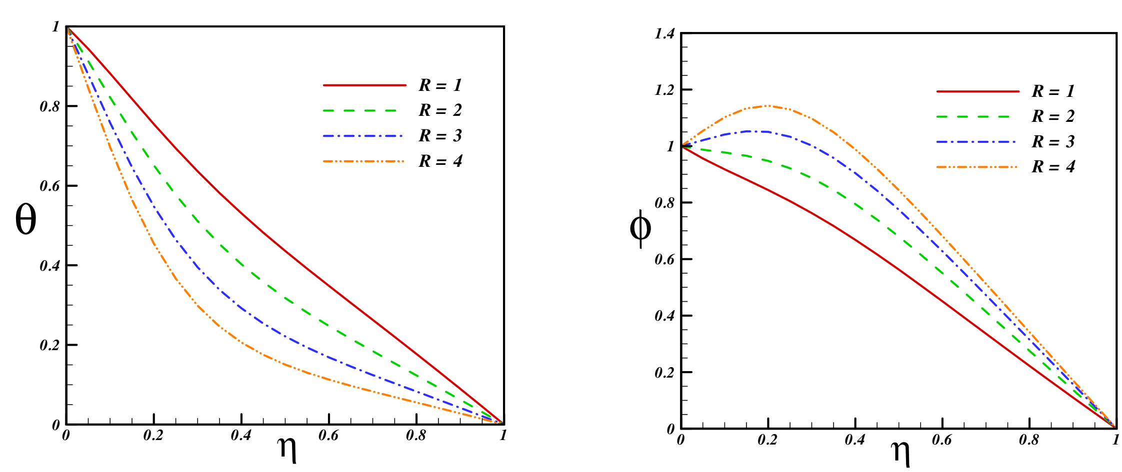


(a) (b)

Fig. 2 Comparison of (a) the temperature profiles between the present work and Mehmood and Ali (2008) when $\lambda = 0.5$, $M = 1$, $R = 0.5$ and $Kr = 0.5$; (b) velocity profile between the present work and (Vajravelu and Kumar (2004)) when $\phi = 0$, $Kr = 0$, $M = 1$ and $\lambda = 1$.

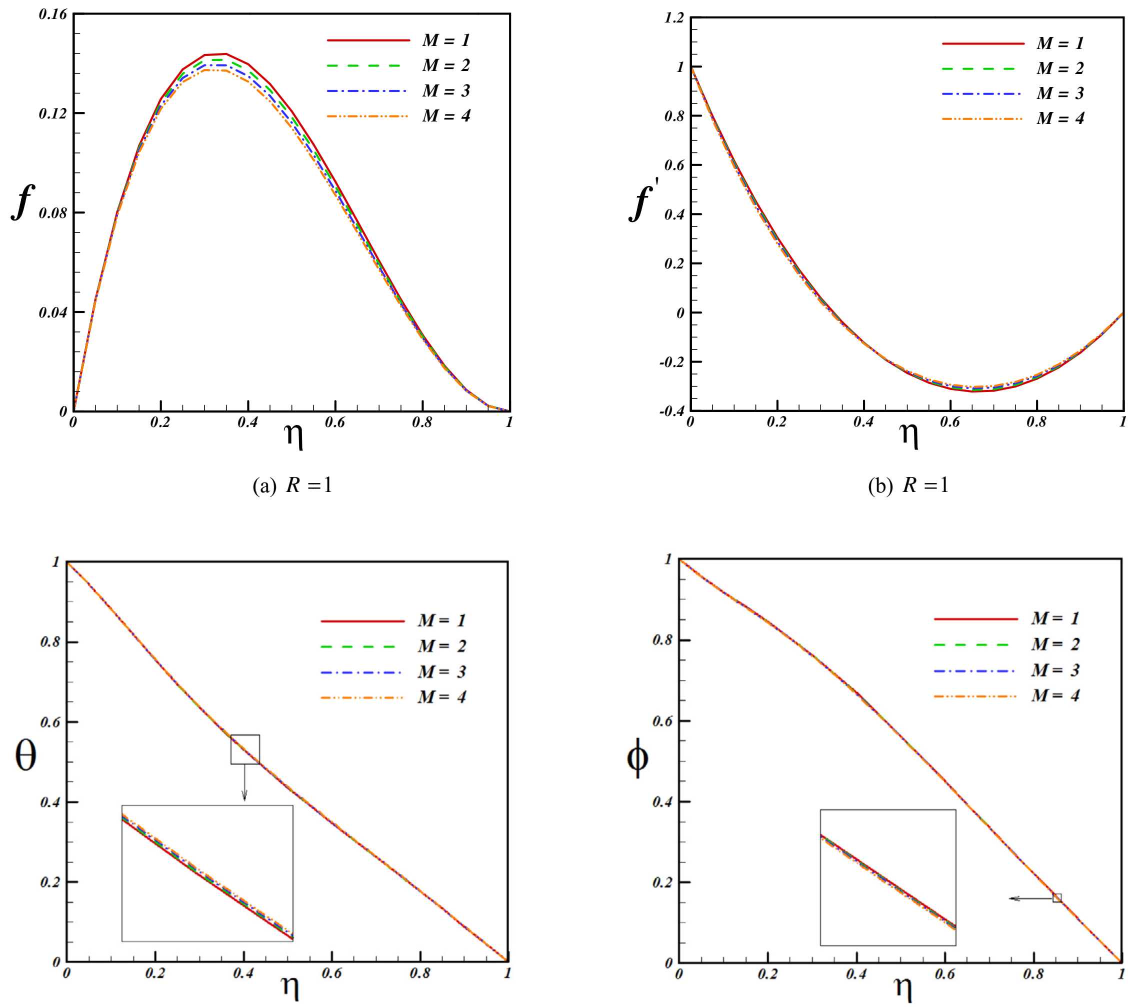


(a) $M = 1$ (b) $M = 1$



(c) $M = 1, Ec = 0.1, Sc = 0.1, Nt = 0.1, Nb = 0.1$ (d) $M = 1, Ec = 0.1, Sc = 0.1, Nt = 0.1, Nb = 0.1$

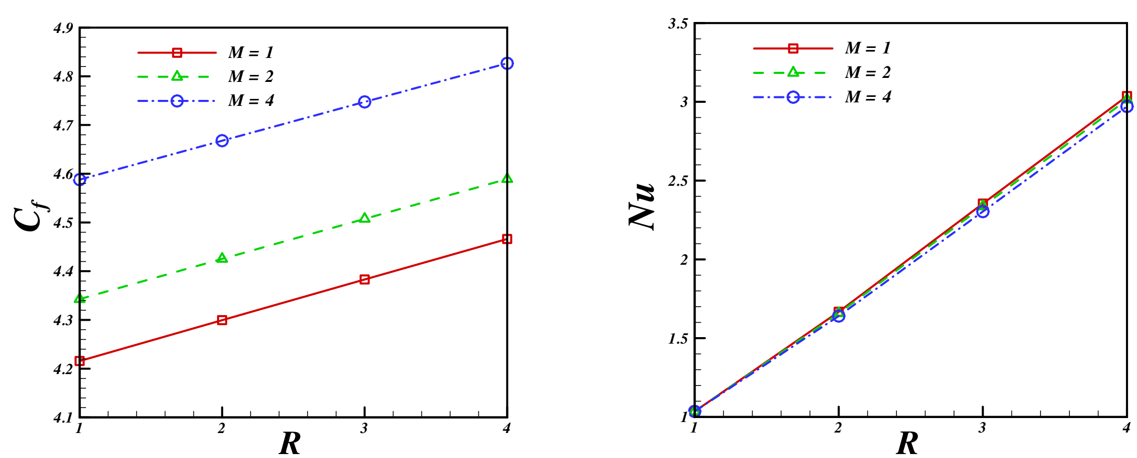
Fig. 3 Effect of viscosity parameter on velocity, temperature and concentration profiles when Pr = 10.



(a) $R = 1$ (b) $R = 1$

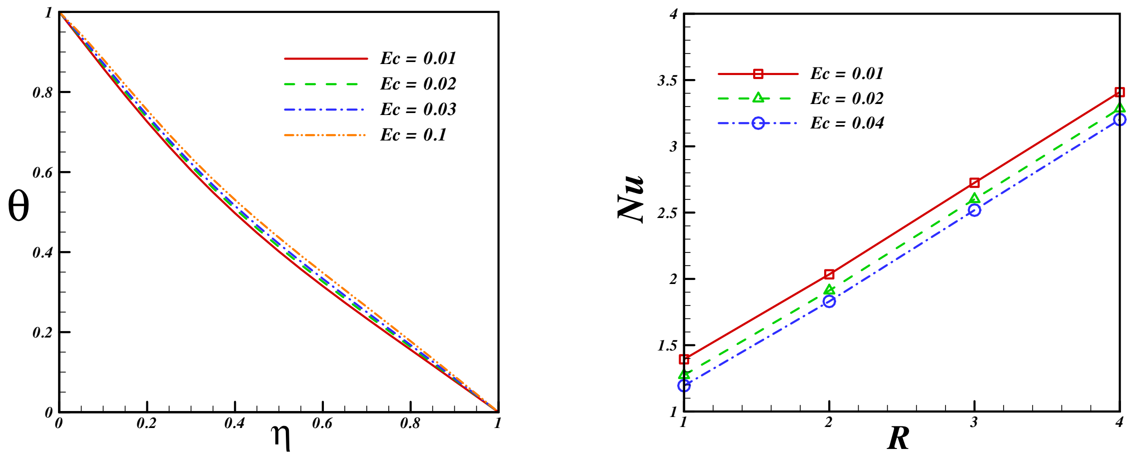
(c) $R = 1, Ec = 0.1, Sc = 0.1, Nt = 0.1, Nb = 0.1$ (d) $R = 1, Ec = 0.1, Sc = 0.1, Nt = 0.1, Nb = 0.1$

Fig. 4 Effect of Magnetic parameter on velocity, temperature and concentration profiles when Pr = 10.



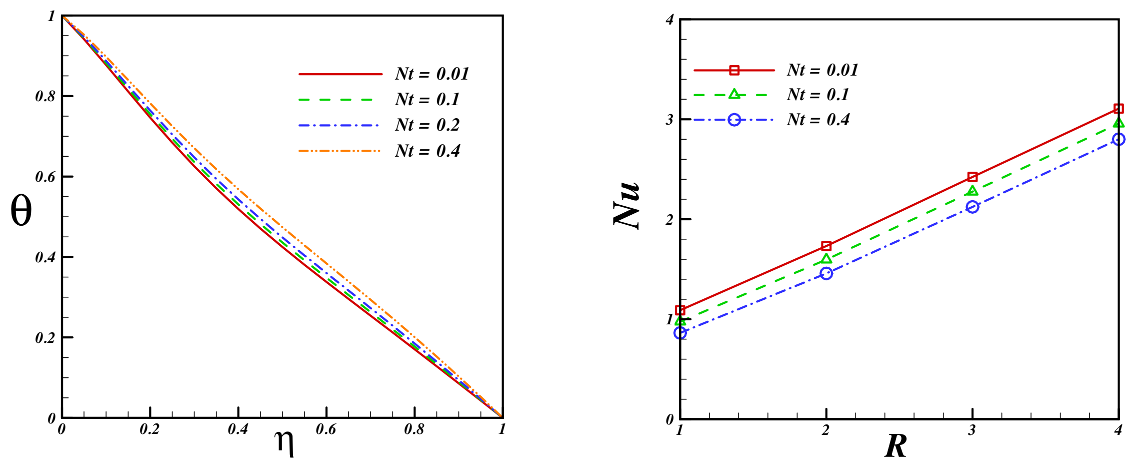
(a) (b) $Ec = 0.1, Sc = 0.1, Nt = 0.1, Nb = 0.1$

Fig. 5 Effects of viscosity and Magnetic parameters on skin friction coefficient and Nusselt number when Pr = 10.



(a) $M = 1, R = 1, Sc = 0.1, Nt = 0.1, Nb = 0.1$ (b) $M = 1, Sc = 0.1, Nt = 0.1, Nb = 0.1$

Fig. 6 Effect of Eckert number on temperature profile and Nusselt number when Pr = 10.



(a) $M = 1, R = 1, Sc = 0.1, Ec = 0.1, Nb = 0.1$ (b) $M = 1, Sc = 0.1, Ec = 0.1, Nb = 0.1$

Fig. 7 Effect of thermophoretic parameter on temperature profile and Nusselt number when Pr = 10.

$$f(\eta) = \sum_{k=0}^{\infty} F(k)(\eta - \eta_0)^k \tag{15}$$

by combining (14) and (15) $f(\eta)$ can be obtained:

$$f(\eta) = \sum_{k=0}^{\infty} \left[\frac{d^k f(\eta)}{d\eta^k} \right]_{\eta=\eta_0} \frac{(\eta - \eta_0)^k}{k!} \tag{16}$$

Eq. (16) implies that the concept of the differential transformation is derived from Taylor's series expansion, but the method does not evaluate the derivatives symbolically. However, relative derivatives are calculated by an iterative procedure that is described by the transformed equations of the original functions. From the definitions of (14) and (15), it is easily proven that the transformed functions comply with the basic mathematical operations shown below. In real applications, the function $f(\eta)$ in (16) is expressed by a finite series and can be written as:

$$f(\eta) = \sum_{k=0}^{N} F(k)(\eta - \eta_0)^k \tag{17}$$

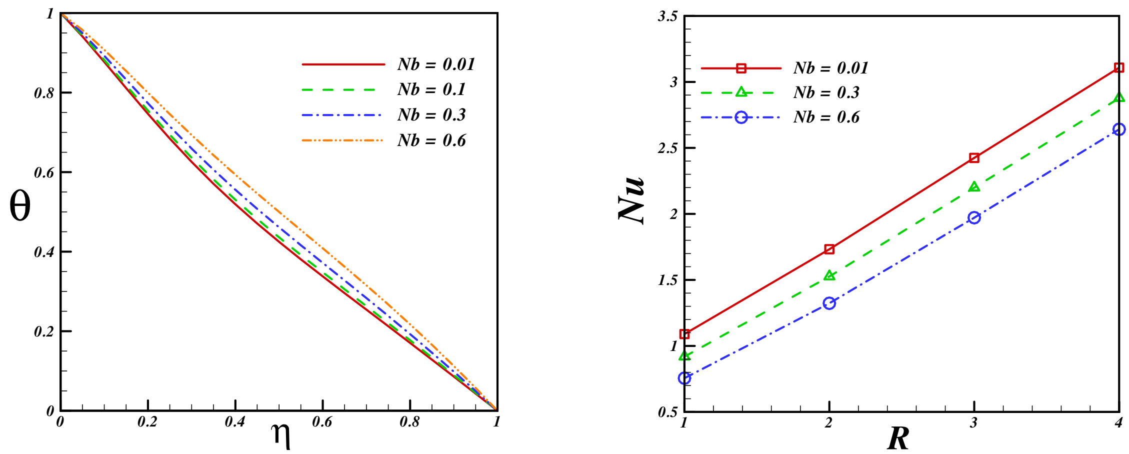
Eq. (16) implies that $f(\eta) = \sum_{k=N+1}^{\infty} (F(k)(\eta - \eta_0)^k)$ is negligibly small, where N is series size.

Theorems to be used in the transformation procedure, which can be evaluated from (14) and (15), are given below (Table 1).

3.2. Solution with Differential Transformation Method

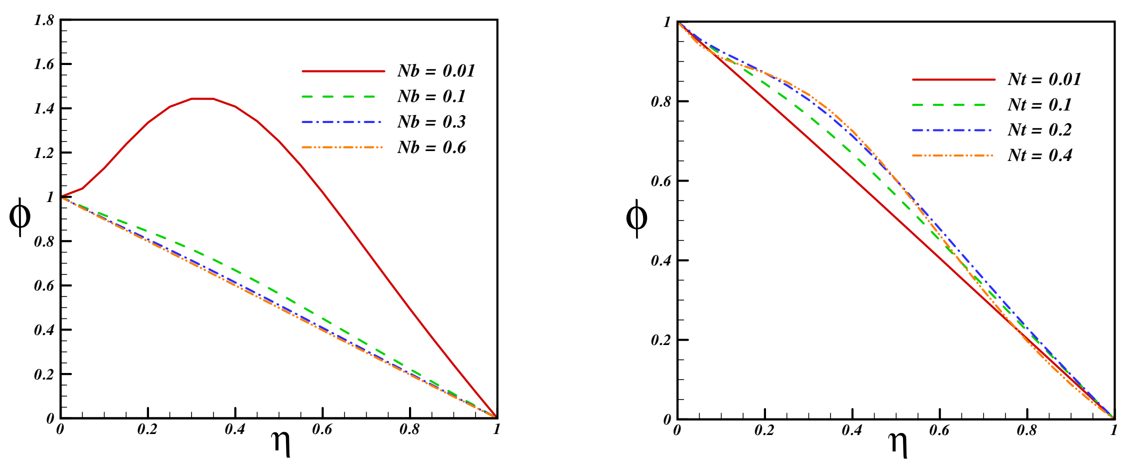
Now Differential Transformation Method has been applied into governing equations (Eqs. 8–10). Taking the differential transforms of Eqs. 8–10 with respect to χ and considering $H = 1$ gives:

$$\begin{aligned}
&(k+1)(k+2)(k+3)(k+4)F[k+4] \\
&\quad - R\left(\sum_{m=0}^{k}((k-m+1)F[k-m+1](m+1)(m+2)F[m+2]) \right. \\
&\quad \left. \times \sum_{m=0}^{k}((k-m)F[k-m](m+1)(m+2)(m+3)F[m+3])\right) \\
&\quad - M(k+1)(k+2)F[k+2] = 0
\end{aligned} \tag{18}$$



(a) $M=1, R=1, Sc=0.1, Nt=0.1, Ec=0.1$ (b) $M=1, Sc=0.1, Nt=0.1, Ec=0.1$

Fig. 8 Effect of Brownian parameter on temperature profile and Nusselt number when Pr = 10.



(a) $Nt=0.1$ (b) $Nb=0.1$

Fig. 9 Effects of Brownian and thermophoretic parameters on concentration profile when $M = 1$, $R = 1$, $Sc = 0.1$, $Ec = 0.1$, $\Pr = 10$.

$$F[0]=0,\quad F[1]=1,\quad F[2]=a_1,\quad F[3]=a_2 \tag{19}$$

$$\begin{aligned}&(k+1)(k+2)\Theta[k+2]+\Pr\left(R\sum_{m=0}^{k}(F[k-m](m+1)\Theta[m+1])\right.\\&\left.+4Ec\frac{A_1}{A_3}\sum_{m=0}^{k}((k-m+1)F[k-m+1](m+1)F[m+1])\right)\\&+Nb\sum_{m=0}^{k}((k-m+1)\Phi[k-m+1](m+1)\Theta[m+1])\\&+Nt\sum_{m=0}^{k}((k-m+1)\Theta[k-m+1](m+1)\Theta[m+1])=0\end{aligned} \tag{20}$$

$$\Theta[0]=1,\quad \Theta[1]=a_3 \tag{21}$$

$$\begin{aligned}&(k+1)(k+2)\Phi[k+2]+Sc\,R\sum_{m=0}^{k}(F[k-m](m+1)\Phi[m+1])\\&+\frac{Nt}{Nb}(k+1)(k+2)\Theta[k+2]=0\end{aligned}$$

$$\Phi[0]=1,\quad \Phi[1]=a_4 \tag{22}$$

where $F[k]$, $\Theta[k]$ and $\Phi[k]$ are the differential transforms of $f(\eta)$, $\theta(\eta)$, $\phi(\eta)$ and a_1, a_2, a_3, a_4 are constants which can be obtained through boundary condition. Solving this problem, it can be expressed;

$$\begin{aligned}&F[0]=0,\quad F[1]=1,\quad F[2]=a_1,\quad F[3]=a_2,\quad F[4]=0,\\&F[5]=\frac{1}{60}R(2a_1^2-3a_2),\ldots\end{aligned} \tag{23}$$

$$\begin{aligned}&\Theta[0]=1,\quad \Theta[1]=a_3,\quad \Theta[2]=-0.5\Pr(4Ec+Nb\,a_3+Nt\,a_3),\\&\Theta[3]=\frac{\Pr}{6}\left(-Ra_3-16Ec\,a_1-2Nb\,a_3a_4+4Nb\Pr Ec+\Pr Nb^2a_3\right.\\&\left.\quad+2Nb\Pr Nt\,a_3-2Nt\,a_3^2+4Nt\Pr Ec+3\Pr Nt^2a_3\right),\ldots\end{aligned} \tag{24}$$

$$\begin{aligned}&\Phi[0]=1,\ \Phi[1]=a_3,\ \Phi[2]=0.5\frac{Nt\Pr(4Ec+Nba_3+Nta_3)}{Nb},\\&\Phi[3]=-\frac{1}{6}(Sc\,R\,a_4Nb-Nt\Pr a_3-16Nt\Pr Ec\,a_1\\&\quad-2Nt\Pr Nb\,a_3a_4+4Nt\Pr^2Nb\,Ec+Nt\Pr^2Nb^2a_3\\&\quad+2Nb\Pr^2Nt^2a_3-2Nt^2a_3\Pr+4\Pr^2Nt^2Ec+\Pr^2Nt^3a_3),\ldots\end{aligned} \tag{25}$$

Substituting Eqs. 23–25 into the main Eq. (18), closed form of the solutions can be obtained:

$$F(\eta) = \eta + a_1\eta^2 + a_2\eta^3 + \left(\frac{1}{60}R\left(2a_1^2 - 3a_2\right)\right)\eta^5 + \cdots \quad (26)$$

$$\theta(\eta) = a_3\eta - 0.5\mathrm{Pr}(4Ec + Nb\,a_3 + Nt\,a_3)\eta^2 + \cdots \quad (27)$$

$$\phi(\eta) = a_4\eta + 0.5\frac{Nt\mathrm{Pr}(4Ec + Nb\,a_3 + Nt\,a_3)}{Nb}\eta^2 + \cdots \quad (28)$$

By substituting the boundary condition from Eq. (11) into Eqs. 26–28 in point $\eta = 1$ it can be obtained the values of a_1, a_2, a_3, a_4. By substituting obtained a_1, a_2, a_3, a_4 into Eqs. 26–28, it can be obtained the expression of $F(\eta)$, $\Theta(\eta)$ and $\Phi(\eta)$.

4. Results and discussion

In this study, DTM is applied to simulate nanofluid flow and heat transfer in the presence of magnetic field. The influences of the viscosity parameter, Magnetic parameter, thermophoretic parameter and Brownian parameter on flow, heat and mass transfer characteristics have been investigated. In order to verify the correctness of the present DTM code, we have compared the results for the temperature profiles with those reported by Mehmood and Ali (2008) when $\phi = 0$ (regular or Newtonian fluid) and also with those of Vajravelu and Kumar (2004) when $Kr = 0$ (non-rotational fluid). This comparison shows a good agreement (Fig. 2).

Figs. 3 and 4 show the effect of viscosity and Magnetic parameters on velocity, temperature and concentration profiles. Effects of viscosity and Magnetic parameters on skin friction coefficient and Nusselt number are shown in Fig. 5. As viscosity parameter increases velocity decreases especially at middle point. Effect of Magnetic parameter on velocity profile is similar to that of viscosity parameter. So, skin friction coefficient is an increasing function of viscosity and Magnetic parameters. Thermal boundary layer thickness decreases with an increase of viscosity parameter but an opposite trend is observed for concentration boundary layer thickness. Increasing Magnetic parameter leads to augment in thermal boundary layer thickness. As shown in Fig. 5, Nusselt number increases with the rise of viscosity parameter while it reduces with the rise of Magnetic parameter. Fig. 6 shows the effect of Eckert number on temperature profile and Nusselt number. Temperature increases with an increase of viscous dissipation. So Nusselt number decreases with an increase of Eckert number.

Effects of thermophoretic and Brownian parameter on temperature profile and Nusselt number are depicted in Figs. 7 and 8. These parameters have a similar effect on temperature profile. It means that temperature increases with augment of these parameters and in turn Nusselt number has reverse relationship with thermophoretic and Brownian parameter. Fig. 9 depicts the effects of Brownian and thermophoretic parameters on concentration profile. As Brownian motion increases concentration of nanofluid decreases while opposite trend is observed for thermophoretic parameter.

5. Conclusion

MHD nanofluid flow, heat and mass transfer between two horizontal parallel plates are investigated to count in the effects of Brownian motion and thermophoresis in the nanofluid model. Differential Transformation Method is used to solve the governing equations. The effects of thermophoretic parameter, Magnetic parameter, Brownian parameter, and viscosity parameter on profiles of concentration, velocity, and temperature are examined. Results show that skin friction increases with an increase of viscosity and Magnetic parameters. Also it can be found that Nusselt number increases with increase of viscosity parameter while it decreases with an increase of Magnetic parameter, thermophoretic parameter and Brownian parameter.

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