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ORIGINAL ARTICLE

A fractional model for dye removal

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Abstract The adsorption process has a fractional property, and a fractional model is suggested to study a transport model of direct textile industry wastewater. An approximate solution of the concentration is obtained by the variational iteration method.

1. Introduction

Textile industries produce huge amounts of polluted effluents that are normally discharged into surface water bodies and groundwater aquifers (Ardejani et al., 2007; Khaled et al., 2009; Gupta and Suhas, 2009; Sinha et al., 2013; Garaje et al., 2013). These wastewaters cause much damage to the ecological system and quality of the surface water obtained and create a lot of disturbance to groundwater resources (Ardejani et al., 2007).

To model the adsorption process of the direct textile industry wastewater, the following transport equation is normally adopted (Ardejani et al., 2007):

$$R\frac{\partial C}{\partial t} + KS\rho_d = 0 \tag{1}$$

where
C = equilibrium concentration of the solution.
S = quantity of mass sorbed on the solid surface (mg/g).
R = the retardation factor.
K = delay constant.
ρ_d = bulk density of the medium (1/1000 mg/mm^3).
Langmuir isotherm reveals the relationship between C and S, which reads (Ardejani et al., 2007).

$$S = \frac{Q_0 K_L C}{1 + K_L C} \tag{2}$$

where
Q_0 = maximum adsorption capacity.
K_L = Langmuir constant.
K_F = partition coefficient indicating adsorption capacity.

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Combining Eqs. (1) and (2) together, we obtain the following nonlinear equation

$$R\frac{\partial C}{\partial t}+\frac{K\rho_d Q_0 K_L C}{1+K_L C}=0,\ C(0)=C_0 \quad (3)$$

It is pointed out, however, that the adsorption process is of fractional property, and can be modeled by a fractional differential equation (Quiroga et al., 2013a,b).

2. Fractional model for dye removal

According to the fractional statistical theory of adsorption (Quiroga et al., 2013a,b), we can modify Eq. (3) in the form:

$$R\frac{\partial^\alpha C}{\partial t^\alpha}+\frac{K\rho_d Q_0 K_L C}{1+K_L C}=0 \quad (4)$$

where the fractional derivative is defined as (He, 2014)

$$\frac{\partial^\alpha C}{\partial t^\alpha}=\frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_{t_0}^{t}(s-t)^{n-\alpha-1}[C_0(s)-C(s)]ds \quad (5)$$

where $C_0(t)$ is a known function.

By the fractional complex transform (Li and He, 2010; He and Li, 2012; Li et al., 2012)

$$T=\frac{t^\alpha}{\Gamma(1+\alpha)} \quad (6)$$

we can convert Eq. (4) into its differential partner, which reads

$$R\frac{\partial C}{\partial T}+\frac{K\rho_d Q_0 K_L C}{1+K_L C}=0 \quad (7)$$

We re-write Eq. (7) in the form

$$\frac{\partial C}{\partial T}+aC+bC\frac{\partial C}{\partial T}=0,\ C(0)=C_0 \quad (8)$$

where

$$a=\frac{K\rho_d Q_0 K_L}{R}$$

$$b=K_L$$

Using the variational iteration method (He and Wu, 2007; He, 2006, 2012), we can construct the following iteration algorithm:

$$C_{n+1}(T)=C_n(T)-\int_0^T e^{a(s-T)}\left(\frac{dC_n(s)}{ds}+aC_n(s)+bC_n(s)\frac{dC_n(s)}{ds}\right)ds \quad (9)$$

or

$$C_{n+1}(T)=C_0(T)-b\int_0^T e^{a(s-T)}C_n(s)\frac{dC_n(s)}{ds}ds \quad (10)$$

Eq. (9) is called the variational iteration algorithm-I, and Eq. (10) the variational iteration algorithm-II. We begin with

$$C=C_0e^{-\beta T} \quad (11)$$

where

β = an unknown constant to be determined later.

By the variational iteration algorithm-II, we have

$$C_1(T)=C_0e^{-\beta T}+b\beta C_0^2\int_0^t e^{a(s-T)}e^{-2\beta s}ds$$
$$=C_0e^{-\beta T}+\frac{b\beta C_0^2}{a-2\beta}(e^{-2\beta T}-e^{-aT}) \quad (12)$$

From Eq. (8), we have an additional initial condition, that is

$$\frac{dC}{dT}(0)=\frac{aC_0}{1+bC_0} \quad (13)$$

Using this relationship, we can identify β in Eq. (12):

$$C_1'(0)=-\beta C_0+\frac{b\beta C_0^2}{a-2\beta}(-2\beta+a)=\frac{aC_0}{1+bC_0} \quad (14)$$

From Eq. (14) β can be solved, which is

$$\beta=\frac{-(abC_0^2-aC_0+2C_0')+\sqrt{(abC_0^2-aC_0+2aC_0')^2+8a^2C_0C_0'(1-bC_0)}}{4C_0(1-bC_0)} \quad (15)$$

We, therefore, obtain the following analytical solution:

$$C(T)=C_0e^{-\beta T}+\frac{b\beta C_0^2}{a-2\beta}(e^{-2\beta T}-e^{-aT}) \quad (16)$$

Finally we have

$$C(t)=C_0\exp\left(-\frac{\beta t^\alpha}{\Gamma(1+\alpha)}\right)+\frac{b\beta C_0^2}{a-2\beta}\left[\exp\left(-\frac{2\beta t^\alpha}{\Gamma(1+\alpha)}\right)-\exp\left(-\frac{at^\alpha}{\Gamma(1+\alpha)}\right)\right] \quad (17)$$

where $a=K\rho_dQ_0K_L/R$ and $b=K_L$.

3. Conclusion

We give a fractional model for dye removal, when the fractional order $\alpha=1$, the fractional model becomes the classic one. The approximate solution reveals that C changes with t^α, and decreases approximately exponentially from C_0 at $t=0$ to a final value $C=0$ when t tends to infinity.

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