Journal of King Saud University (Science) (2011) 23, 35–39

King Saud University
Journal of King Saud University (Science)
www.ksu.edu.sa
www.sciencedirect.com

ORIGINAL ARTICLE

Modified HPM for solving systems of Volterra integral equations of the second kind

Jafar Biazar *, Mostafa Eslami

Department of Mathematics, Faculty of Sciences, University of Guilan, P.O. Box 413351914, P.C. 4193833697, Rasht, Iran

Received 28 May 2010; accepted 10 June 2010
Available online 13 June 2010

KEYWORDS
New technique;
Systems of Volterra integral equations

Abstract In this paper, a new reliable technique for solving systems of Volterra integral equations of the second kind has been introduced. This new method is resulted from HPM by a simple modification. This modification is based on the existence of Taylor expansion of the kernel and source terms. To illustrate the new modification on HPM some examples are presented. Comparisons of the results of applying modified HPM and classical HPM reveal the new technique is very effective and convenient.

1. Introduction

Homotopy perturbation method established by He, on 1998 (He, 1999, 2000). The ability of the method will be more appear when it is applied to solve nonlinear equations (Siddiqui et al., 2006; Cveticanin, 2006; Biazar et al., 2009; Abbasbandy, 2006; Biazar and Ghazvini, 2009; Biazar et al., 2007; Ozis and Yildirim, 2007; Ghori et al., 2007; Rana et al., 2007; Tari et al., 2007; Ariel et al., 2006; Odibat and Momani, 2008). In this article a simple modification on the method will be studied and will be applied to solve systems of Volterra integral equations of the second kind.

A system of Volterra integral equations of the second kind (Delves and Mohamed, 1985) can be considered as

$$F(t) = G(t) + \int_0^t K(s,t,F(s))ds, \tag{1}$$

where

$$F(t) = (f_1(t),\ f_2(t), \dots,\ f_n(t))^T,$$
$$G(t) = (g_1(t), g_2(t), \dots, g_n(t))^T,$$
$$K(s,t,F(s)) = (k_1(s,t,F(s)), k_2(s,t,F(s)), \dots, k_n(s,t,F(s)))^T.$$

If $k(s,t,F(s))$, be linear, system (1) can be presented as the following simple form:

$$F_i(t) = g_i(t) + \int_0^t \sum_{j=1}^n k_{i,j}(s,t)F_j(s)ds, \quad i = 1,2,\dots,n. \tag{2}$$

Using the usual vector-matrix notation, this linear system will be written as

* Corresponding author. Tel.: +98 9113251508.
E-mail addresses: biazar@guilan.ac.ir, jafar.biazar@gmail.com (J. Biazar), m.eslami@guilan.ac.ir, eslami_mostafa@yahoo.com (M. Eslami).

1018-3647 © 2010 King Saud University. All rights reserved. Peer-review under responsibility of King Saud University.
doi:10.1016/j.jksus.2010.06.004

Production and hosting by Elsevier

$$F(t) = G(t) + \int_0^t K(s,t)F(s)ds. \quad (3)$$

For solving system (2), by He's homotopy perturbation method we construct the following homotopy

$$\phi_i(t) = g_i(t) + p\int_0^t \sum_{j=1}^{n} k_{i,j}(s,t)\phi_j(s)ds, \quad i = 1,2,\ldots,n. \quad (4)$$

Suppose the solutions of system (4) have the following form

$$\phi_i(t) = \phi_{i,0}(t) + p\phi_{i,1}(t) + p^2\phi_{i,2}(t) + \cdots, \quad i = 1,2,\ldots,n, \quad (5)$$

where $\phi_{i,j}, i = 1,2,\ldots,n$, are functions which should be determined.

Substituting (5) into (4) and equating the coefficients of p with the same power leads to

$$p^0 : \phi_{i,0}(t) = g_i(t), \quad i = 1,2,\ldots,n,$$

$$p^1 : \phi_{i,1}(t) = \int_0^t \sum_{j=1}^{n} k_{i,j}(s,t)\phi_{j,0}(s)ds, \quad i = 1,2,\ldots,n,$$

$$p^2 : \phi_{i,2}(t) = \int_0^t \sum_{j=1}^{n} k_{i,j}(s,t)\phi_{j,1}(s)ds, \quad i = 1,2,\ldots,n,$$

$$p^3 : \phi_{i,3}(t) = \int_0^t \sum_{j=1}^{n} k_{i,j}(s,t)\phi_{j,2}(s)ds, \quad i = 1,2,\ldots,n,$$

$$\vdots$$

The approximated solutions of (2), therefore, can be obtained by setting $p = 1$

$$F_i(t) = \lim_{p\to 1}\phi_i(t) = \sum_{j=0}^{\infty}\phi_{i,j}(t), \quad i = 1,2,\ldots,n. \quad (6)$$

2. The new technique

To accelerate the convergence of homotopy perturbation method, when it is used for systems of Volterra integral equations of the second kind, if the kernels $k_{i,j}(s,t)$ are separable, say $k_{i,j}(s,t) = k_{i,j,1}(s)k_{i,j,2}(t)$, and functions $k_{i,j,1}(s)$, $k_{i,j,2}(t)$ and $g_i(t)$ are analytic, the new idea is based on the replacement of these functions by their Taylor expansions

$$g_i(t) = \sum_{l=0}^{\infty} g_{i,l}(t), \quad k_{i,j}(s,t) = k_{i,j,1}(s)k_{i,j,2}(t) = \sum_{l=0}^{\infty} k_{i,j,1,l}(s)\sum_{l=0}^{\infty} k_{i,j,2,l}(t). \quad (7)$$

With $g_{i,l}(t) = \frac{g_i(t_l)(t-t_l)}{l!}, k_{i,j,1,l}(t) = \frac{k_{i,j,1}(t_l)(t-t_l}{l!}$, and $k_{i,j,1,l}(s) = \frac{k_{i,j,1}(s_l)(s-s_l)}{l!}$, respectively.

Substitution Eqs. (7) into Eq. (2) results in

$$L(F_j) = F_j(t) - \sum_{l=0}^{\infty} g_{i,l}(t) - \int_0^x \sum_{l=0}^{\infty} k_{i,j,1,l}(s)\sum_{l=0}^{\infty} k_{i,j,2,l}(t)F_j(s)ds = 0, \quad j = 1,2,\ldots,n.$$

The following homotopy can be constructed

$$H(\phi_j, p) = \phi_j(t) - \sum_{l=0}^{\infty} g_{i,l}(t)p^l - p\int_0^x \sum_{l=0}^{\infty} k_{i,j,1,l}(s)p^l \times \sum_{l=0}^{\infty} k_{i,j,2,l}(t)p^l\phi_j(s)ds = 0, \quad j = 1,2,\ldots,n. \quad (8)$$

Substituting (5) into (8), and equating the coefficients of the terms with identical powers of p, components of series solution (6) will be obtained.

This technique is simple and very effective tool which usually leads to the exact solutions. This method can be used for problems that the homotopy perturbation method does not work.

3. Existence and uniqueness of the solution

To prove existence and uniqueness, we extend the same results for the linear Volterra integral equation of the second kind (Linz, 1985), to (3), and use the classical approach, so-called the Picard method. This consists of the following simple iterations:

$$F_n(t) = G(t) + \int_0^t K(t,s)F_{n-1}(s)ds, \quad (9)$$

with

$$F_0(t) = G(t).$$

For simplicity it is convenient to introduce

$$\Psi_n(t) = F_n(t) - F_{n-1}(t), \quad n = 1,2,\ldots, \quad (10)$$

with

$$\Psi_0(t) = G(t).$$

On subtracting from (9) the same equation with n replaced by $n-1$, we obtain

$$\Psi_n(t) = \int_0^t K(t,s)\Psi_{n-1}(s)ds, \quad n = 1,2,\ldots \quad (11)$$

Also from (4),

$$F_n(t) = \sum_{i=0}^{n}\Psi_i(t). \quad (12)$$

In the following theorem, we use this iteration to prove the existence and uniqueness of the solution under the hypothesis that $K(t,s)$ and $G(t)$ are continues.

Theorem 1. *If $G(t)$ and $K(t,s)$ are continuous in $0 \leqslant s \leqslant t \leqslant T$, then the system (3) has a unique continuous solution for $0 \leqslant t \leqslant T$.*

Proof 1. There exist constants g and k such that

$$\|G(t)\| \leqslant g, \quad 0 \leqslant t \leqslant T, \quad \|K(t,s)\| \leqslant k, \quad 0 \leqslant s \leqslant t \leqslant T.$$

We first prove, by induction, that

$$\|\Psi_n\| \leqslant \frac{g(kt)^n}{n!} \quad 0 \leqslant t \leqslant T \quad n = 1,2,\ldots \quad (13)$$

Let's assume validity of (13) for $n-1$, then form (11),

$$\|\Psi_n\| \leqslant \frac{gk^n}{(n-1)!}\int_0^t s^{n-1}ds = \frac{gk^n t^n}{n!}.$$

Since (11) is obviously true for $n = 0$, it holds for all n. These bounds make it obvious that the sequence $F_n(t)$, in (6) convergence uniformly and we can write

$$F(t) = \sum_{i=0}^{\infty}\Psi_i(t). \quad (14)$$

We now show that $F(t)$ satisfies Eq. (3). By uniform convergence of (14), order of integration and summation in the following expression, can be changed

$$\int_0^t K(t,s)\sum_{i=0}^{\infty}\Psi_i(s)ds = \sum_{i=0}^{\infty}\int_0^t K(t,s)\Psi_i(s)ds = \sum_{i=0}^{\infty}\Psi_{i+1}(t) = \sum_{i=0}^{\infty}\Psi_i(t) - G(t).$$

This proves that $F(t)$, defined by (14), satisfies Eq. (3). Each of the $\Psi_i(t)$ are clearly continuous. Therefore $F(t)$ is continuous, since it is the limit of a uniformly convergent series of continuous functions.

To show that $F(t)$ is the unique continuous solution, suppose that there exists another continuous solution $\widetilde{F}(t)$, then

$$F(t) - \widetilde{F}(t) = \int_0^t K(t,s)(F(s) - \widetilde{F}(s))ds. \tag{15}$$

Since $F(t)$ and $\widetilde{F}(t)$ are both continuous there exists a constant B such that

$$\|F(t) - \widetilde{F}(t)\| \leqslant B, \quad 0 \leqslant t \leqslant T.$$

Substituting this into (9) gives,

$$\|F(t) - \widetilde{F}(t)\| \leqslant kBt, \quad 0 \leqslant t \leqslant T.$$

Repeating this substitution leads to

$$\|F(t) - \widetilde{F}(t)\| \leqslant \frac{B(kt)^n}{n!}, \quad 0 \leqslant t \leqslant T,$$

for any n. obviously $\frac{B(kt)^n}{n!} \to 0$, as $n \to \infty$ for any t, which implies that

$$F(t) = \widetilde{F}(t), \quad 0 \leqslant t \leqslant T.$$

This completes the proof. □

4. Numerical Example

In this part three examples are provided. These examples are considered to illustrate ability and reliability of the new technique.

Example 1. Consider the following linear system of Volterra integral equations of the second kind

$$\begin{cases} u(x) = f(x) + \int_0^x (te^t v(t) + u(t))dt, \\ v(x) = g(x) + \int_0^x (-te^{-t}u(t) - v(t))dt, \end{cases} \tag{16}$$

where $f(x) = 1 - \frac{x^2}{2}$, and $g(x) = 1 + \frac{x^2}{2}$.

Homotopy perturbation method:

Using HPM, leads to

$$\begin{cases} u(x) = f(x) + p\int_0^x (te^t v(t) + u(t))dt, \\ v(x) = g(x) + p\int_0^x (-te^{-t}u(t) - v(t))dt. \end{cases} \tag{17}$$

Substituting (5) into (17), and equating the coefficients of the terms with identical powers of p, the following terms will be achieved

$$\begin{cases} u_0(x) = 1 - \frac{x^2}{2}, \\ v_0(x) = 1 + \frac{x^2}{2}, \end{cases}$$

$$\begin{cases} u_1(x) = 4 + x - \frac{1}{6}x^3 - 4e^x + 4xe^x - \frac{3}{2}x^2e^x + \frac{1}{2}x^3e^x, \\ v_1(x) = 2 - x - \frac{1}{6}x^3 - 2e^{-x} - 2xe^{-x} - \frac{3}{2}x^2e^{-x} - \frac{1}{2}x^3e^{-x}, \end{cases}$$

$$\vdots$$

Then the series solution, by the homotopy perturbation method, is as follows:

$$\begin{cases} u(x) = \sum_{i=0}^{\infty} u_i(x) = 1 - \frac{x^2}{2} + 4 + x - \frac{1}{6}x^3 - 4e^x + 4xe^x \\ \qquad - \frac{3}{2}x^2e^x + \frac{1}{2}x^3e^x + 22 + \cdots \\ v(x) = \sum_{i=0}^{\infty} v_i(x) = 1 + \frac{x^2}{2} + 2 - x - \frac{1}{6}x^3 - 2e^{-x} - 2xe^{-x} \\ \qquad - \frac{3}{2}x^2e^{-x} - \frac{1}{2}x^3e^{-x} + 8 + \cdots \end{cases}$$

The new technique:

We use the Taylor series for te^t and $-te^{-t}$

$$te^t = \sum_{n=0}^{\infty}\frac{t^{n+1}}{n!},$$

$$-te^{-t} = \sum_{n=0}^{\infty}\frac{(-1)^{n+1}t^{n+1}}{n!}.$$

And construct the following homotopy, after substitution of Taylor series in Eq. (16)

$$\begin{cases} u(x) = 1 - p\frac{x^2}{2} + p\int_0^x \left(\sum_{n=0}^{\infty}\frac{t^{n+1}}{n!}p^n v(t) + u(t)\right)dt, \\ v(x) = 1 + p\frac{x^2}{2} + p\int_0^x \left(\sum_{n=0}^{\infty}\left(\frac{(-1)^{n+1}t^{n+1}}{n!}p^n u(t) - v(t)\right)\right)dt. \end{cases} \tag{18}$$

Substituting (5) into (18), and equating of the terms with identical powers of p, gives

$$p^0 : \begin{cases} u_0(x) = 1, \\ v_0(x) = 1, \end{cases}$$

$$p^1 : \begin{cases} u_1(x) = x, \\ v_1(x) = -x, \end{cases}$$

$$\vdots$$

$$p^{j+1} : \begin{cases} u_{j+1}(x) = \int_0^x \left(\sum_{k=0}^{j}\left(\frac{t^{k+1}}{(k)!}v_{j-k}(t)\right) + u_j(t)\right)dt, \\ v_{j+1}(x) = \int_0^x \left(\sum_{k=0}^{j}\left(-\frac{(-1)^k t^{k+1}}{k!}u_{j-k}(t)\right) - v_j(t)\right)dt, \end{cases}$$

$$\vdots$$

Therefore the solution of Example 1 can be readily presented by

$$u(x) = \sum_{i=0}^{\infty} u_i(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots$$

$$v(x) = \sum_{i=0}^{\infty} v_i(x) = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \cdots$$

With the summation, $u(x) = e^x$, $v(x) = e^{-x}$, which is exact solution.

Example 2. Let's solve the following system of integral equation ($|x| < 1$)

$$\begin{cases} u(x) - 1 = \int_0^x \left(\frac{1}{2(t+1)} u(t) - 2tv(t)\right) dt, \\ v(x) = \int_0^x \left(\frac{1}{2(t+1)} v(t) + 2tu(t)\right) dt. \end{cases} \tag{19}$$

Homotopy perturbation method:

Using HPM, we have

$$\begin{cases} u(x) - 1 = p \int_0^x \left(\frac{1}{2(t+1)} u(t) - 2tv(t)\right) dt, \\ v(x) = p \int_0^x \left(\frac{1}{2(t+1)} v(t) + 2tu(t)\right) dt. \end{cases} \tag{20}$$

Substituting (5) into (20), and equating the coefficients of the terms with identical powers of p, leads to

$$\begin{cases} u_0(x) = 1, \\ v_0(x) = 0, \end{cases}$$

$$\begin{cases} u_1(x) = \frac{1}{2}\ln(1+x), \\ v_1(x) = x^2, \end{cases}$$

$$\vdots$$

Therefore the approximation solution of Example 2 can be written as

$$\begin{cases} u(x) = \sum_{i=0}^{\infty} u_i(x) = 1 + \frac{1}{2}\ln(1+x) - \frac{1}{2}x^4 + \frac{1}{8}\ln(1+x) + \cdots \\ v(x) = \sum_{i=0}^{\infty} v_i(x) = x^2 + \frac{1}{2}x^2\ln(1+x) + \cdots \end{cases}$$

The new technique:

The Taylor series of the function $\frac{1}{2(x+1)}$, can be presented as follows:

$$\frac{1}{2(x+1)} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2},$$

by substitution of these series into Eq. (19) the following homotopy can be constructed.

$$\begin{cases} u(x) - 1 = p \int_0^x \left(\sum_{n=0}^{\infty}\left(\frac{(-1)^n t^n}{2} p^n u(t) - 2tv(t)\right)\right) dt, \\ v(x) = p \int_0^x \left(\sum_{n=0}^{\infty}\left(\frac{(-1)^n t^n}{2} p^n v(t) + 2tu(t)\right)\right) dt. \end{cases} \tag{21}$$

Substituting (5) into (21), and equating the coefficients of the terms with identical powers of p, reads to

$$p^0 : \begin{cases} u_0(x) = 1, \\ v_0(x) = 0, \end{cases}$$

$$p^1 : \begin{cases} u_1(x) = \frac{1}{2}x, \\ v_1(x) = x^2, \end{cases}$$

$$\vdots$$

$$p^{j+1} : \begin{cases} u_{j+1}(x) = \int_0^x \left(\sum_{k=0}^{j}\left(\frac{(-1)^k t^k}{2} u_{j-k}(t)\right) - 2tv_j(t)\right) dt, \\ v_{j+1}(x) = \int_0^x \left(\sum_{k=0}^{j}\left(\frac{(-1)^k t^k}{2} v_{j-k}(t)\right) + 2tu_j(t)\right) dt, \end{cases}$$

$$\vdots$$

Therefore the solution of Example 2, can be readily presented as the following

$$u(x) = \sum_{i=0}^{\infty} u_i(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{69}{128}x^4 + \cdots$$

$$v(x) = \sum_{i=0}^{\infty} v_i(x) = x^2 + \frac{1}{2}x^3 - \frac{1}{8}x^4 + \frac{1}{16}x^5 - \frac{79}{384}x^6 - \cdots$$

And hence,

$$\begin{cases} u(x) = \sqrt{1+x}\cos(x^2), \\ v(x) = \sqrt{1+x}\sin(x^2). \end{cases}$$

Which is solution of Example 2.

Example 3. Consider the following system of integral equations

$$\begin{cases} f_1(t) + \int_0^t e^{-(s-t)} f_1(s)ds + \int_0^t \cos(s-t) f_2(s)ds = \cosh t + t\sin t, \\ f_2(t) + \int_0^t e^{s+t} f_1(s)ds + \int_0^x t\cos s f_2(s)ds = 2\sin t + t(\sin^2 t + e^t), \end{cases} \tag{22}$$

where $f_1(t) = e^{-t}$, and $f_2(t) = 2\ \sin t$ (Biazar and Ghazvini, 2009).

Let's use the new technique to solve this equation

Substitution of Taylor series of $f_i(t)$ and $k_{i,j}(s,t)$ in the Eq. (22), the following homotopy can be constructed

$$\begin{cases} f_1(t) = -p\int_0^t \sum_{n=0}^{\infty}\frac{(-s)^n}{n!}p^n \sum_{n=0}^{\infty}\frac{t^n}{n!}p^n f_1(s)ds \\ \quad -p\int_0^t \begin{pmatrix} \sum_{n=0}^{\infty}(-1)^n\frac{s^{2n}}{(2n)!}p^n \sum_{n=0}^{\infty}(-1)^n\frac{t^{2n}}{(2n)!}p^n f_2(s) \\ +\sum_{n=0}^{\infty}(-1)^n\frac{s^{2n+1}}{(2n+1)!}p^n \sum_{n=0}^{\infty}(-1)^n\frac{t^{2n+1}}{(2n+1)!}p^n f_2(s) \end{pmatrix} ds \\ \quad +\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{t^n}{n!}+\frac{(-t)^n}{n!}\right)p^n + t\sum_{n=0}^{\infty}(-1)^n\frac{t^{2n+1}}{(2n+1)!}p^n, \\ f_2(t) = -p\int_0^t \sum_{n=0}^{\infty}\frac{(s)^n}{n!}p^n \sum_{n=0}^{\infty}\frac{t^n}{n!}p^n f_1(s)ds - p\int_0^t t\sum_{n=0}^{\infty}(-1)^n\frac{s^{2n}}{(2n)!}p^n f_2(s)ds \\ \quad +2\sum_{n=0}^{\infty}(-1)^n\frac{t^{2n+1}}{(2n+1)!}p^n + t\left(\frac{1}{2}-\frac{1}{2}\sum_{n=0}^{\infty}(-1)^n\frac{(2t)^{2n}}{(2n)!}p^n + \sum_{n=0}^{\infty}\frac{(t)^n}{n!}p^n\right). \end{cases} \tag{23}$$

Substituting (5) into (23), and equating the terms with identical powers of p, leads to

$$p^0 : \begin{cases} f_{1,0}(t) = 1 + t^2, \\ f_{2,0}(t) = 3t, \end{cases}$$

$$p^1 : \begin{cases} f_{1,1}(t) = -\frac{7}{6}t^4 - t - \frac{1}{3}t^3 - \frac{3}{2}t^2, \\ f_{2,1}(t) = -\frac{7}{6}t^3 + t^2 - t, \end{cases}$$

$$p^2 : \begin{cases} f_{1,2}(t) = \frac{61}{120}t^6 + t^2 - \frac{1}{60}t^5 + \frac{7}{4}t^4 + \frac{1}{6}t^3, \\ f_{2,2}(t) = \frac{7}{12}t^5 + \frac{3}{2}t^3 - \frac{5}{6}t^4 - t^2, \end{cases}$$

$$\vdots$$

$$p^j : \begin{cases} f_{1,j}(t) = -\int_0^t \sum_{i=0}^{j-1}\sum_{k=0}^{j-i-1}\frac{(-s)^i}{i!}\frac{t^k}{k!}f_{1,j-k-i-1}(s)ds \\ \quad -\int_0^t \begin{pmatrix} \sum_{i=0}^{j-1}\sum_{k=0}^{j-i-1}(-1)^i\frac{s^{2i}}{(2i)!}(-1)^k\frac{t^{2k}}{(2k)!}f_{2,j-k-i-1}(s) \\ +\sum_{i=0}^{j-1}\sum_{k=0}^{j-i-1}(-1)^i\frac{s^{2i+1}}{(2i+1)!}(-1)^k\frac{t^{2k+1}}{(2k+1)!}f_{2,j-k-i-1}(s) \end{pmatrix} ds \\ \quad +\frac{1}{2}\left(\frac{t^j}{j!}+\frac{(-t)^j}{j!}\right) + t(-1)^j\frac{t^{2j+1}}{(2j+1)!}, \\ f_{2,j}(t) = -\int_0^t \sum_{i=0}^{j-1}\sum_{k=0}^{j-i-1}\frac{(s)^i}{i!}\frac{t^k}{k!}f_{1,j-k-i-1}(s)ds - \int_0^t t\sum_{k=0}^{j-1}(-1)^k\frac{s^{2k}}{(2k)!}f_{2,j-k-1}(s)ds \\ \quad +2(-1)^j\frac{t^{2j+1}}{(2j+1)!} + t\left(-\frac{1}{2}(-1)^j\frac{(2t)^{2j}}{(2j)!}+\frac{(t)^j}{j!}\right), \end{cases}$$

$$\vdots$$

The series form of the solution is given by

$$f_1(t) = \sum_{i=0}^{\infty} f_{1,i}(t) = 1 - t + \frac{t^2}{2} - \frac{t^3}{6} + \frac{t^4}{24} + \cdots$$

$$f_2(t) = \sum_{i=0}^{\infty} f_{2,i}(t) = 2t - 2\frac{(t)^3}{6} + 2\frac{(t)^5}{120} + \cdots$$

With the closed form, $f_1(t) = e^{-t}$, and $f_2(t) = 2\ \sin t$, which is exact solution.

5. Conclusion

In this paper, a modified form of HPM, for solving systems of Volterra integral equations of the second kind, is studied successfully. This new idea is based on the series forms of the function $G(t)$ and the kernel $K(s,t)$. So it is necessary to mention that this procedure can be used when $G(t)$, $K(s,t)$ are analytic. The most important note which is worth to mention is that this procedure leads, almost, to exact solution for both linear and nonlinear equations. The computations associated with examples were performed using the package maple 13.

References

Abbasbandy, S., 2006. Application of the integral equations: homotopy perturbation method and Adomian's decomposition method. Applied Mathematics and Computation 173, 493–500.

Ariel, P.D., Hayat, T., Asghar, S., 2006. Homotopy perturbation method and axisymmetric flow over a stretching sheet. International Journal of Nonlinear Science and Numerical Simulation 7 (4), 399–406.

Biazar, J., Ghazvini, H., 2009. He's homotopy perturbation method for solving systems of Volterra integral equations of the second kind. Chaos, Solitons and Fractals 2, 770–777.

Biazar, J., Eslami, M., Ghazvini, H., 2007. Homotopy perturbation method for systems of partial differential equations. International Journal of Nonlinear Science and Numerical Simulation 8 (3), 411–416.

Biazar, J., Ghazvini, H., Eslami, M., 2009. He's homotopy perturbation method for systems of integro-differential equations. Chaos, Solitons and Fractals 39 (3), 1253–1258.

Cveticanin, L., 2006. Homotopy – perturbation method for pure nonlinear differential equation. Chaos, Solitons and Fractals 30, 1221–1230.

Delves, L.M., Mohamed, J.L., 1985. Computational Methods for Integral Equation. Cambridge University press, Cambridge.

Ghori, Q.K., Ahmed, M., Siddiqui, A.M., 2007. Application of homotopy perturbation method to squeezing flow of a Newtonian fluid. International Journal of Nonlinear Science and Numerical Simulation 8 (2), 179–184.

He, J.H., 1999. Homotopy perturbation technique. Computer Methods in Applied Mechanics and Engineering 178, 257–262.

He, J.H., 2000. A coupling method of homotopy technique and perturbation technique for nonlinear problems. International Journal of Non-Linear Mechanics 35 (1), 37–43.

Linz, P., 1985. Analytical and Numerical Method for Volterra Equations. SIAM.

Odibat, Z., Momani, S., 2008. Modified homotopy perturbation method: application to quadratic Riccati differential equation of fractional order. Chaos, Solitons and Fractals 36 (1), 167–174.

Ozis, T., Yildirim, A., 2007. A note on He's homotopy perturbation method for van der Pol oscillator with very strong nonlinearity. Chaos, Solitons and Fractals 34 (3), 989–991.

Rana, M.A., Siddiqui, A.M., Ghori, Q.K., 2007. Application of He's homotopy perturbation method to Sumudu transform. International Journal of Nonlinear Science and Numerical Simulation 8 (2), 185–190.

Siddiqui, A.M., Zeb, A., Ghori, Q.K., 2006. Homotopy perturbation method for thin film flow of a fourth grade fluid down a vertical cylinder. Physics Letters A 352, 404–410.

Tari, H., Ganji, D.D., Rostamian, M., 2007. Approximate solutions of K (2,2), KdV and modified KdV equations by variational iteration method, homotopy perturbation method and homotopy analysis method. International Journal of Nonlinear Science and Numerical Simulation 8 (2), 203–210.