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Traveling wave solutions for some important coupled nonlinear physical models via the coupled Higgs equation and the Maccari system

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Abstract In this article, the $\exp(-\Phi(\xi))$-expansion method has been successfully implemented to seek traveling wave solutions of the coupled Higgs field equation and the Maccari system. The result reveals that the method together with the first order ordinary differential equation is a very influential and effective tool for solving coupled nonlinear partial differential equations in mathematical physics and engineering. The obtained solutions have been articulated by the hyperbolic functions, trigonometric functions and rational functions with arbitrary constants. Numerical results together with the graphical representation explicitly reveal the high efficiency and reliability of the proposed algorithm.

1. Introduction

The investigation of exact traveling wave solutions to nonlinear partial differential equations (NPDEs) plays an important role in the study of nonlinear physical models. NPDEs are widely used to describe a variety of complex phenomena in the field of applied sciences, such as, solid mechanics, propagation of shallow water waves, long wave and chemical reaction–diffusion models, fluid dynamics, biophysics, Higgs mechanism, quantum field theory, plasma physics, etc. However, it is more difficult to solve the NPDEs. Many researchers have tried to search direct methods that can solve the NPDEs easily. In the recent years, with the development of ansatz concept, some methods for finding analytic solutions to NPDEs, for instance, the homotopy analysis method (Liao, 2005, 2009), the three-wave method (Darvishi and Najafi, 2012a), the extended homoclinic test approach (Darvishi and Najafi, 2012b), the improved F-expansion method (Wang and Zhang, 2005), the projective Riccati equation method (Yan, 2003), the Weierstrass elliptic function method (Kudryashov, 1990), the Jacobi elliptic function expansion method (Chen and Wang, 2005; Liu et al., 2001), the tanh-function method (Wazwaz, 2008; Abdou, 2007; Malfliet, 2004) and the homotopy perturbation method (Chun and Sakthivel, 2010; Mohyud-Din et al., 2011), the

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inverse scattering transform method (Ablowitz and Clarkson, 1991), Hirota's bilinear method (Hirota, 1971), the Exp-function method (He and Wu, 2006; Akbar and Ali, 2011, 2012; Mohyud-Din et al., 2010; Noor et al., 2008, 2010), the variational iteration method (Mohyud-Din et al., 2009), the truncated Painleve expansion method (Kudryashov, 1991), the Kudryashov method (Kim et al., 2014), the extended tanh-method (Abdou and Soliman, 2006; El-Wakil and Abdou, 2007), the (G'/G)-expansion method (Kim and Sakthivel, 2012), the homogeneous balance method (Zhao et al., 2006; Zhaosheng, 2004; Zhao and Tang, 2002; Wang and Li, 2008; Wang et al., 2010a,b, 2012), the $\exp(-\Phi(\xi))$-expansion (Akbar and Ali, 2014; Uddin et al., 2014; Rahman et al., 2014) and other methods (Wazwaz, 2009; Belgacem et al., 2013; Lee and Sakthivel, 2013; Bahrami et al., 2011; Manafian and Zamanpour, 2013a,b; Ma et al., 2008; Mohyud-Din et al., 2011; Sakthivel et al., 2010) have been developed and used for searching the exact solutions. The aim of this article is to find exact traveling solutions of the coupled Higgs field equation and the Maccari system by using the $\exp(-\Phi(\xi))$-expansion (Akbar and Ali, 2014; Uddin et al., 2014; Rahman et al., 2014). In the studied literature this method has not been applied to the above mentioned equations. The advantage of the proposed method over the (G'/G)-expansion method is that it provides new exact traveling wave solutions along with additional free parameters. The exact solutions have their great importance to reveal the internal mechanism of the physical phenomena. Apart from the physical importance, the close-form solutions of nonlinear evolution equations assist the numerical solvers to compare the correctness of their results and help them in the stability analysis. Soliton, periodic traveling wave solution, kink, and cuspons are originated when the related physical parameters received their particular values. Algebraic manipulation of the proposed scheme with the help of Maple is much easier than the other methods.

The article is arranged as follows: Section 2 presents the description of the $\exp(-\Phi(\xi))$ expansion method. Section 3 is devoted to deriving the traveling wave solutions of the coupled Higgs equation and the Maccari system. The physical explanations and comparisons of the solutions are presented in Section 4 and Section 5 respectively. Conclusions have been drawn in Section 6.

2. Description of the $\exp(-\Phi(\xi))$-expansion method

Let us consider a general nonlinear PDE in the form

$$F(u, u_t, u_x, u_y, u_{xx}, u_{yy}, u_{tt}, u_{tx}, \ldots), \tag{1}$$

where, $u = u(x, y, t)$ is an unknown function, F is a polynomial in $u(x, y, t)$ and its derivative in which highest order derivatives and nonlinear terms are involved and the subscripts stand for the partial derivatives. The main steps of this method are as follows:

Step 1: Combine the real variables x, y and t by a compound variable ξ

$$u(x, y, t) = u(\xi),\ \ \xi = x + y \pm ct, \tag{2}$$

where, c is the speed of the traveling wave. The traveling wave transformation (2), converts Eq. (1) into an ordinary differential equation (ODE) for $u = u(\xi)$:

$$\Re(u, u', u'', u''', \cdots), \tag{3}$$

where, $\Re$ is a polynomial of u and its derivatives and the superscripts indicate the ordinary derivatives with respect to ξ.

Step 2: Suppose the traveling wave solution of Eq. (3) can be expressed as follows:

$$u(\xi) = \sum_{i=0}^{N} A_i(\exp(-\Phi(\xi)))^i, \tag{4}$$

where, A_i $(0 \leqslant i \leqslant N)$ are constants to be determined, such that $A_N \neq 0$ and $\Phi = \Phi(\xi)$ satisfies the following ordinary differential equation:

$$\Phi'(\xi) = \exp(-\Phi(\xi)) + \mu \exp(\Phi(\xi)) + \lambda, \tag{5}$$

Eq. (5) gives the following solutions:

Family 1: When $\mu \neq 0$, $\lambda^2 - 4\mu > 0$,

$$\Phi(\xi) = \ln\left(\frac{-\sqrt{(\lambda^2 - 4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi + E)\right) - \lambda}{2\mu}\right) \tag{6}$$

Family 2: When $\mu \neq 0$, $\lambda^2 - 4\mu < 0$,

$$\Phi(\xi) = \ln\left(\frac{\sqrt{(4\mu - \lambda^2)}\tan\left(\frac{\sqrt{(4\mu-\lambda^2)}}{2}(\xi + E)\right) - \lambda}{2\mu}\right) \tag{7}$$

Family 3: When $\mu = 0$, $\lambda \neq 0$, and $\lambda^2 - 4\mu > 0$,

$$\Phi(\xi) = -\ln\left(\frac{\lambda}{\exp(\lambda(\xi + E)) - 1}\right) \tag{8}$$

Family 4: When $\mu \neq 0$, $\lambda \neq 0$, and $\lambda^2 - 4\mu = 0$,

$$\Phi(\xi) = \ln\left(-\frac{2(\lambda(\xi + E) + 2)}{\lambda^2(\xi + E)}\right) \tag{9}$$

Family 5: When $\mu = 0$, $\lambda = 0$, and $\lambda^2 - 4\mu = 0$,

$$\Phi(\xi) = \ln(\xi + E) \tag{10}$$

$A_N, \cdots\cdots, V, \lambda, \mu$ are constants to be determined latter, $A_N \neq 0$. The positive integer N can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms occurring in Eq. (3).

Step 3: Substitute Eq. (4) into Eq. (3) and then we account the function $\exp(-\Phi(\xi))$. As a result of this substitution, we get a polynomial of $\exp(-\Phi(\xi))$. We equate all the coefficients of same power of $\exp(-\Phi(\xi))$ to zero. This procedure yields a system of algebraic equations whichever can be solved to find $A_1, A_2 \cdots\cdots, V, \lambda, \mu$ with the aid of Maple. Substituting the values of $A_1, A_2 \cdots\cdots, V, \lambda, \mu$ into Eq. (4) along with general solutions of Eq. (5) completes the determination of the solution of Eq. (1).

3. Applications to some important nonlinear coupled physical models

In this section, we will apply the $\exp(-\Phi(\xi))$-expansion method to find the exact traveling wave solutions of the coupled Higgs equation and the Maccari system (Lee and

Sakthivel, 2013; Bahrami et al., 2011; Manafian and Zamanpour, 2013a,b).

3.1. The coupled Higgs equation

Let us consider the following coupled Higgs equation

$$\left.\begin{aligned} &u_{tt}-u_{xx}+|u|^2u-2u\,v=0\\ &v_{tt}+v_{xx}-\left(|u|^2\right)_{xx}=0\end{aligned}\right\} \tag{11}$$

The Higgs field Eq. (11) describes a system of conserved scalar nucleon interaction with a neutral scalar meson. Here, v represents a real scalar meson field and u a complex scalar nucleon field. Eq. (11) is related to some nonlinear models with physical interests. Using the traveling wave variables

$$u(x,t)=e^{i\,\omega}U(\xi),\; v(x,t)=V(\xi), \tag{12}$$

where, $\omega=px+rt$ and $\xi=x+c\,t$, the coupled Higgs Eq. (11) can be reduced to the ODE of the form

$$\left.\begin{aligned} &(c^2-1)U''+(p^2-r^2)\,U-2UV+U^3=0\\ &(c^2+1)\,V''-(U^2)''=0\end{aligned}\right\} \tag{13}$$

Integrating the second equation in system (13) and neglecting the constant of integration we find,

$$V=\frac{U^2}{(c^2+1)}. \tag{14}$$

Substituting (14) into the first equation of (13), we obtain

$$(c^4-1)U''+(c^2+1)\,(p^2-r^2)\,U+(c^2-1)\,U^3=0, \tag{15}$$

where, primes denote differentiation with respect to ξ. By balancing the height order derivative term U'' with the nonlinear term U^3 in (15), gives $N=1$. Therefore, the $\exp(-\Phi(\xi))$-expansion method allows us to use the solution in the following form:

$$U(\xi)=A_0+A_1\exp(-\Phi(\xi)),\quad A_1\neq 0 \tag{16}$$

By substituting (5) and (16) into the Eq. (15) and equating the coefficients of $(\exp(-\Phi(\xi)))^i$, $(i=0,1,2,3,4,\cdots)$ to zero, we obtain a system of algebraic equations (for simplicity the algebraic equations are not displayed here). Solving the obtained system by using Maple, the following sets of solutions are obtained:

$$c=1,\; p=\pm r,\; r=r,\; A_0=A_0,\; A_1=A_1, \tag{17}$$

$$c=-1,\; p=\pm r,\; r=r,\; A_0=A_0,\; A_1=A_1, \tag{18}$$

$$c=c,\, p=\pm\sqrt{\frac{-4c^2\mu+\lambda^2c^2+2r^2-\lambda^2+4\mu}{2}},$$

$$A_0=\pm\frac{\lambda\sqrt{2c^22}}{2},\; A_1=\mp\sqrt{2(c^2+1)}, \tag{19}$$

where λ, μ and r are arbitrary constants.

According to Eqs. (17) and (18), the traveling wave solutions of the coupled Higgs Eq. (11) with the help of Eqs. (12) and (14) are obtained in the following form:

when $\lambda^2 4\mu>0$, $\mu\neq 0$, we have

$$\left.\begin{aligned} &u_1(x,t)=e^{\pm i\,r(x\pm t)}\times\left\{A_0-A_1\left(\frac{2\mu}{\sqrt{\Omega}\tanh(\frac{\sqrt{\Omega}}{2}(x+t+E))+\lambda}\right)\right\},\\ &v_1(x,t)=\tfrac{1}{2}\times\left\{A_0-A_1\left(\frac{2\mu}{\sqrt{\Omega}\tanh(\frac{\sqrt{\Omega}}{2}(x+t+E))+\lambda}\right)\right\}^2\end{aligned}\right. \tag{20}$$

and

$$\left.\begin{aligned} &u_2(x,t)=e^{\pm i\,r(x\pm t)}\times\left\{A_0-A_1\left(\frac{2\mu}{\sqrt{\Omega}\tanh(\frac{\sqrt{\Omega}}{2}(x-t+E))+\lambda}\right)\right\},\\ &v_2(x,t)=\tfrac{1}{2}\times\left\{A_0-A_1\left(\frac{2\mu}{\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2}(x-t+E)\right)+\lambda}\right)\right\}^2\end{aligned}\right. \tag{21}$$

where, $\Omega=\lambda^2-4\mu$ and E is arbitrary constant.

When $\lambda^2-4\mu<0$, we have

$$\left.\begin{aligned} &u_3(x,t)=e^{\pm\,ir(x\pm t)}\times\left\{A_0+A_1\left(\frac{2\mu}{\sqrt{\Omega}\tan(\frac{\sqrt{\Omega}}{2}(x+t+E))-\lambda}\right)\right\}\\ &v_3(x,t)=\tfrac{1}{2}\times\left\{A_0+A_1\left(\frac{2\mu}{\sqrt{\Omega}\tan(\frac{\sqrt{\Omega}}{2}(x+t+E))-\lambda}\right)\right\}^2\end{aligned}\right. \tag{22}$$

and

$$\left.\begin{aligned} &u_4(x,t)=e^{\pm\,ir(x\pm t)}\times\left\{A_0+A_1\left(\frac{2\mu}{\sqrt{\Omega}\tan(\frac{\sqrt{\Omega}}{2}(x-t+E))-\lambda}\right)\right\}\\ &v_4(x,t)=\tfrac{1}{2}\times\left\{A_0+A_1\left(\frac{2\mu}{\sqrt{\Omega}\tan(\frac{\sqrt{\Omega}}{2}(x-t+E))-\lambda}\right)\right\}^2\end{aligned}\right. \tag{23}$$

where, $\Omega=4\mu-\lambda^2$ and E is arbitrary constant.

When $\lambda^2-4\mu>0$, $\mu=0$, we have

$$\left.\begin{aligned} &u_5(x,t)=e^{\pm i\,r(x\pm t)}\times\left\{A_0+A_1\left(\frac{\lambda}{\exp(\lambda(x+t+E))-1}\right)\right\},\\ &v_5(x,t)=\tfrac{1}{2}\times\left\{A_0+A_1\left(\frac{\lambda}{\exp(\lambda(x+t+E))-1}\right)\right\}^2\end{aligned}\right. \tag{24}$$

and

$$\left.\begin{aligned} &u_6(x,t)=e^{\pm i\,r(x\pm t)}\times\left\{A_0+A_1\left(\frac{\lambda}{\exp(\lambda(x-t+E))-1}\right)\right\},\\ &v_6(x,t)=\tfrac{1}{2}\times\left\{A_0+A_1\left(\frac{\lambda}{\exp(\lambda(x-t+E))-1}\right)\right\}^2\end{aligned}\right. \tag{25}$$

where E is arbitrary constant.

When $\mu\neq 0$, $\lambda\neq 0$, and $\lambda^2-4\mu=0$, we have

$$\left.\begin{aligned} &u_7(x,t)=e^{\pm i\,r(x\pm t)}\times\left\{A_0-A_1\left(\frac{\lambda^2(x+t+E)}{2(\lambda(x+t+E)+2)}\right)\right\},\\ &v_7(x,t)=\tfrac{1}{2}\times\left\{A_0-A_1\left(\frac{\lambda^2(x+t+E)}{2(\lambda(x+t+E)+2)}\right)\right\}^2\end{aligned}\right. \tag{26}$$

and

$$\left.\begin{aligned} &u_8(x,t)=e^{\pm i\,r(x\pm t)}\times\left\{A_0-A_1\left(\frac{\lambda^2(x-t+E)}{2(\lambda(x-t+E)+2)}\right)\right\},\\ &v_8(x,t)=\tfrac{1}{2}\times\left\{A_0-A_1\left(\frac{\lambda^2(x-t+E)}{2(\lambda(x-t+E)+2)}\right)\right\}^2\end{aligned}\right. \tag{27}$$

where, E is arbitrary constant.

When $\mu=0$, $\lambda=0$, and $\lambda^2-4\mu=0$, we have

$$u_9(x,t)=e^{\pm\,ir(x\pm t)}\times\left\{A_0+\frac{A_1}{x+t+E}\right\},$$

$$v_9(x,t)=\frac{1}{2}\times\left\{A_0+\frac{A_1}{x+t+E}\right\}^2 \tag{28}$$

and

$$u_{10}(x,t) = e^{\pm ir(x\pm t)} \times \left\{A_0 + \frac{A_1}{x-t+E}\right\},$$
$$v_{10}(x,t) = \frac{1}{2} \times \left\{A_0 + \frac{A_1}{x-t+E}\right\}^2 \quad (29)$$

Again, due to Eq. (19), the exact traveling wave solutions of the coupled Higgs Eq. (11) are also as follows:

when $\lambda^2 - 4\mu > 0$, $\mu \neq 0$, we have

$$u_{11}(x,t) = e^{i(px+rt)}\left\{\pm\frac{\lambda\sqrt{-2c^2-2}}{2} \mp \sqrt{-2c^2-2}\left(\frac{2\mu}{\sqrt{\Omega}\tanh(\frac{\sqrt{\Omega}}{2}(x+ct+E))+\lambda}\right)\right\},$$
$$v_{11}(x,t) = \frac{1}{(c^2+1)}\left\{\pm\frac{\lambda\sqrt{-2c^2-2}}{2} \mp \sqrt{-2c^2-2}\left(\frac{2\mu}{\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2}(x+ct+E)\right)+\lambda}\right)\right\}^2 \quad (30)$$

where $\Omega = \lambda^2 - 4\mu$, $p = \pm\sqrt{\frac{-4c^2\mu+\lambda^2c^2+2r^2-\lambda^2+4\mu}{2}}$ and E is arbitrary constant.

When $\lambda^2 - 4\mu < 0$, we have

$$u_{12}(x,t) = e^{i(px+rt)}\left\{\pm\frac{\lambda\sqrt{-2c^2-2}}{2} \pm \sqrt{-2c^2-2}\left(\frac{2\mu}{\sqrt{\Omega}\tan(\frac{\sqrt{\Omega}}{2}(x+ct+E))-\lambda}\right)\right\}$$
$$v_{12}(x,t) = \frac{1}{(c^2+1)}\left\{\pm\frac{\lambda\sqrt{-2c^2-2}}{2} \pm \sqrt{-2c^2-2}\left(\frac{2\mu}{\sqrt{\Omega}\tan\left(\frac{\sqrt{\Omega}}{2}(x+ct+E)\right)-\lambda}\right)\right\}^2 \quad (31)$$

where $\Omega = 4\mu - \lambda^2$, $p = \pm\sqrt{\frac{-4c^2\mu+\lambda^2c^2+2r^2-\lambda^2+4\mu}{2}}$ and E is arbitrary constant.

When $\lambda^2 - 4\mu > 0$, $\mu = 0$, we have

$$u_{13}(x,t) = e^{i(px+rt)}\left\{\pm\frac{\lambda\sqrt{-2c^2-2}}{2} \pm \sqrt{-2c^2-2}\left(\frac{\lambda}{\exp(\lambda(x+ct+E))-1}\right)\right\},$$
$$v_{13}(x,t) = \frac{1}{(c^2+1)}\left\{\pm\frac{\lambda\sqrt{-2c^2-2}}{2} \pm \sqrt{-2c^2-2}\left(\frac{\lambda}{\exp(\lambda(x+ct+E))-1}\right)\right\}^2 \quad (32)$$

where $p = \pm\sqrt{\frac{-4c^2\mu+\lambda^2c^2+2r^2-\lambda^2+4\mu}{2}}$, E is arbitrary constant.

When $\mu \neq 0$, $\lambda \neq 0$, and $\lambda^2 - 4\mu = 0$, we have

$$u_{14}(x,t) = e^{i(px+rt)}\left\{\pm\frac{\lambda\sqrt{-2c^2-2}}{2} \mp \sqrt{-2c^2-2}\left(\frac{\lambda^2(x+ct+E)}{2(\lambda(x+ct+E)+2)}\right)\right\},$$
$$v_{14}(x,t) = \frac{1}{(c^2+1)}\left\{\pm\frac{\lambda\sqrt{-2c^2-2}}{2} \mp \sqrt{-2c^2-2}\left(\frac{\lambda^2(x+ct+E)}{2(\lambda(x+ct+E)+2)}\right)\right\}^2 \quad (33)$$

where $p = \pm\sqrt{\frac{-4c^2\mu+\lambda^2c^2+2r^2-\lambda^2+4\mu}{2}}$, E is arbitrary constant.

When $\mu = 0$, $\lambda = 0$, and $\lambda^2 - 4\mu = 0$, we have

$$u_{15}(x,t) = e^{ir(px+rt)}\left\{\pm\frac{\lambda\sqrt{-2c^2-2}}{2} \pm \frac{\sqrt{-2c^2-2}}{x+ct+E}\right\},$$
$$v_{15}(x,t) = \frac{1}{(c^2+1)}\left\{\pm\frac{\lambda\sqrt{-2c^2-2}}{2} \pm \frac{\sqrt{-2c^2-2}}{x+ct+E}\right\}^2 \quad (34)$$

where $p = \pm\sqrt{\frac{-4c^2\mu+\lambda^2c^2+2r^2-\lambda^2+4\mu}{2}}$, E is arbitrary constant.

The solutions obtained from (20)–(34) are useful and will help the researchers to understand the scalar nucleon interaction with the neutral scalar meson.

3.2. The Maccari system

Let us consider the (2 + 1)-dimensional coupled integrable nonlinear system in the following form

$$\left.\begin{array}{l} iu_t + u_{xx} + uv = 0 \\ v_t + v_y + (|u|^2)_x = 0 \end{array}\right\}, \quad (35)$$

This system brings nonlinear evolution equations that are frequently used to describe location in a small part of space, and motion of the isolated waves in varied fields, such as, nonlinear optics, fluid mechanics, quantum field theory, and plasma physics.

If we apply the following transformation

$$u(x,y,t) = e^{i\omega}U(\xi), \; v(x,y,t) = V(\xi), \quad (36)$$

where $\omega = px + qy + rt$ and $\xi = x + y + ct$, the Maccari system in (35) can be reduced to a system of ODE form as follows:

$$\left.\begin{array}{l} U'' - (r+p^2)U + UV = 0 \\ (c+1)V' + (U^2)' = 0 \end{array}\right\}. \quad (37)$$

Integrating the second equation in (37) and neglecting the constant of integration we find

$$V = -\frac{1}{(c+1)}U^2. \quad (38)$$

Substituting (38) into the first equation of the system (37), we obtain

$$(c+1)U'' - (c+1)(r-p^2)U - U^3 = 0, \quad (39)$$

where, primes denote differentiation with respect to ξ. Balancing the highest order linear term U'' with the nonlinear of the highest order U^3 yields $N = 1$. Therefore, we obtain

$$U(\xi) = A_0 + A_1\exp(-\Phi(\xi)), \quad A_1 \neq 0 \quad (40)$$

By substituting (5) and (40) into the Eq. (39) and equating the coefficient of $(\exp(-\Phi(\xi)))^i$, $(i = 0, 1, 2, 3, 4, \cdots)$ to zero, we obtain a system of algebraic equations. Solving the obtained system, we obtained the following set of solutions:

$$c = -1 + \frac{1}{2}A_1^2, \; p = p, \; q = q, \; r = 2\mu - \frac{1}{2}\lambda^2 + p^2,$$
$$A_0 = \frac{\lambda}{2}A_1, \; A_1 = A_1, \quad (41)$$

where λ and μ are arbitrary constants.

Therefore, the traveling wave solutions for the Maccari system with the help of Eqs. (36) and (38) according to (41) are obtained in the following form:

When $\lambda^2 - 4\mu > 0$, $\mu \neq 0$, we have

$$u_{16}(x,y,t) = A_1\exp\left[i\{px + qy + \left(2\mu - \tfrac{1}{2}\lambda^2 + p^2\right)t\}\right] \times\left\{\frac{\lambda}{2} - \left(\frac{2\mu}{\sqrt{\Omega}\tanh(\frac{\sqrt{\Omega}}{2}(\xi+E))+\lambda}\right)\right\},$$
$$v_{16}(x,y,t) = -\frac{A_1^2}{(c+1)} \times \left\{\frac{\lambda}{2} - \left(\frac{2\mu}{\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2}(\xi+E)\right)+\lambda}\right)\right\}^2 \quad (42)$$

where $\Omega = \lambda^2 - 4\mu$, $\xi = x + y + (\frac{A_1^2}{2} - 1)\,t$ and E is arbitrary constant.

When $\lambda^2 - 4\mu < 0$, we have

$$u_{17}(x,y,t) = A_1\exp\left[i\{px + qy + (2\mu - \tfrac{1}{2}\lambda^2 + p^2)t\}\right] \times\left\{\frac{\lambda}{2} + \left(\frac{2\mu}{\sqrt{\Omega}\tan\left(\frac{\sqrt{\Omega}}{2}(\xi+E)\right)+\lambda}\right)\right\}$$
$$v_{17}(x,y,t) = -\frac{A_1^2}{(c+1)} \times \left\{\frac{\lambda}{2} + \left(\frac{2\mu}{\sqrt{\Omega}\tan\left(\frac{\sqrt{\Omega}}{2}(\xi+E)\right)+\lambda}\right)\right\}^2 \quad (43)$$

where $\Omega = 4\mu - \lambda^2$, $\xi = x + y + (\frac{A_1^2}{2} - 1)\,t$ and E is arbitrary constant.

When $\lambda^2 - 4\mu > 0$, $\mu = 0$, we have

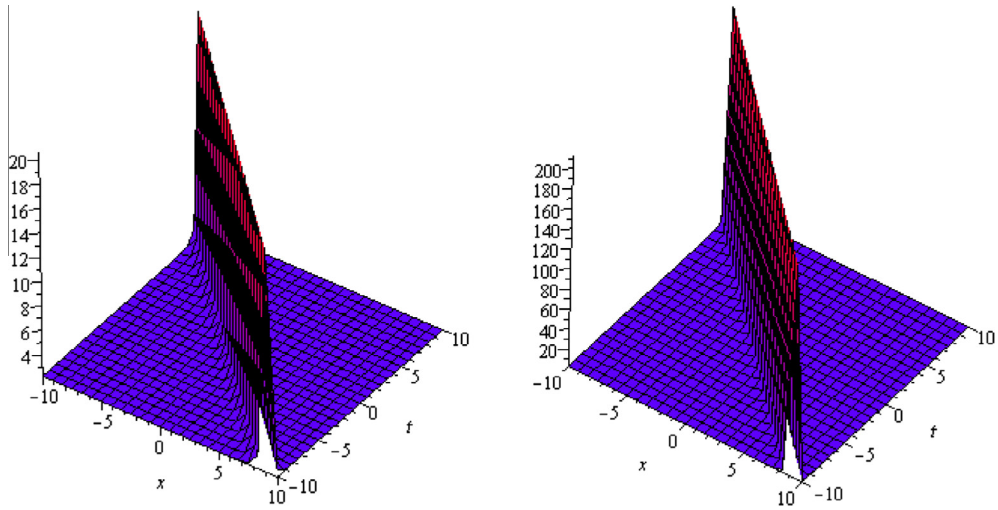

Figure 1 3D plot of soliton solution of u_1 and v_1 with $-10 \leqslant x,t \leqslant 10$ respectively.

$$u_{18}(x,y,t) = A_1 \exp\left[i\{px + qy + (2\mu - \tfrac{1}{2}\lambda^2 + p^2)t\}\right] \times\left\{\tfrac{\lambda}{2} + \left(\tfrac{\lambda}{\exp(\lambda(\xi+E))-1}\right)\right\}, \tag{44}$$

$$v_{18}(x,y,t) = -\tfrac{A_1^2}{(c+1)} \times \left\{\tfrac{\lambda}{2} + \left(\tfrac{\lambda}{\exp(\lambda(\xi+E))-1}\right)\right\}^2$$

where $\Omega = 4\mu - \lambda^2$, $\xi = x + y + (\frac{A_1^2}{2} - 1)\,t$ and E is arbitrary constant.

When $\mu \neq 0$, $\lambda \neq 0$, and $\lambda^2 - 4\mu = 0$, we have

$$u_{19}(x,y,t) = A_1 \exp\left[i\{px + qy + (2\mu - \tfrac{1}{2}\lambda^2 + p^2)t\}\right] \times\left\{\tfrac{\lambda}{2} - \left(\tfrac{\lambda^2(\xi+E)}{2(\lambda(\xi+E)+2)}\right)\right\}, \tag{45}$$

$$v_{19}(x,y,t) = -\tfrac{A_1^2}{(c+1)} \times \left\{\tfrac{\lambda}{2} - \left(\tfrac{\lambda^2(\xi+E)}{2(\lambda(\xi+E)+2)}\right)\right\}^2$$

where $\xi = x + y + (\frac{A_1^2}{2} - 1)\,t$ and E is arbitrary constant.

When $\mu = 0$, $\lambda = 0$, and $\lambda^2 - 4\mu = 0$, we have

$$u_{20}(x,y,t) = A_1 \exp\left[i\{px + qy + (2\mu - \tfrac{1}{2}\lambda^2 + p^2)t\}\right] \times\left\{\tfrac{\lambda}{2} + \tfrac{1}{\xi+E}\right\}, \tag{46}$$

$$v_{20}(x,y,t) = -\tfrac{A_1^2}{(c+1)} \times \left\{\tfrac{\lambda}{2} + \tfrac{1}{\xi+E}\right\}^2$$

where $\xi = x + y + (\frac{A_1^2}{2} - 1)\,t$ and E is arbitrary constant.

4. Physical explanations

Solutions u_1, u_2, v_1, v_2, u_{11}, v_{11}, v_{13}, v_{14} of the coupled Higgs equation and u_{16}, v_{16}, u_{18}, v_{18}, v_{19} of the Maccari system for $y = 0$ are the soliton solutions. Solitons are special kinds of solitary waves. Solitons have a remarkable property that keeps its identity upon interacting with other solitons. Soliton solutions have particle-like structures, for example, magnetic monopoles, and extended structures, like, domain walls and cosmic strings, that have implications in cosmology of the early universe. Fig. 1 shows the shape of the exact soliton solution of u_1 and v_1 for $r = 1$, $c = 1$, $\lambda = 1$, $\mu = -1$, $A_0 = 1$, $A_1 = 2$ and $E = 0.5$ with $-10 \leqslant x, t \leqslant 10$ respectively .The other figures are ignored for simplicity.

Solutions u_3, v_3, u_4, v_4, v_{12} of the coupled Higgs equation and u_{17}, v_{17} of the Maccari system with $y = 0$ represent the exact periodic traveling wave solutions. Periodic solutions are traveling wave solutions that are periodic, such as cos $(x - t)$. Fig. 2 below shows the periodic solution of u_3 and v_3 for $r = 1$, $c = 1$, $\lambda = 2$, $\mu = 3$, $A_0 = 1$, $A_1 = 2$ and $E = 0.5$ with $-10 \leqslant x, t \leqslant 10$ respectively. The other figures are ignored for convenience.

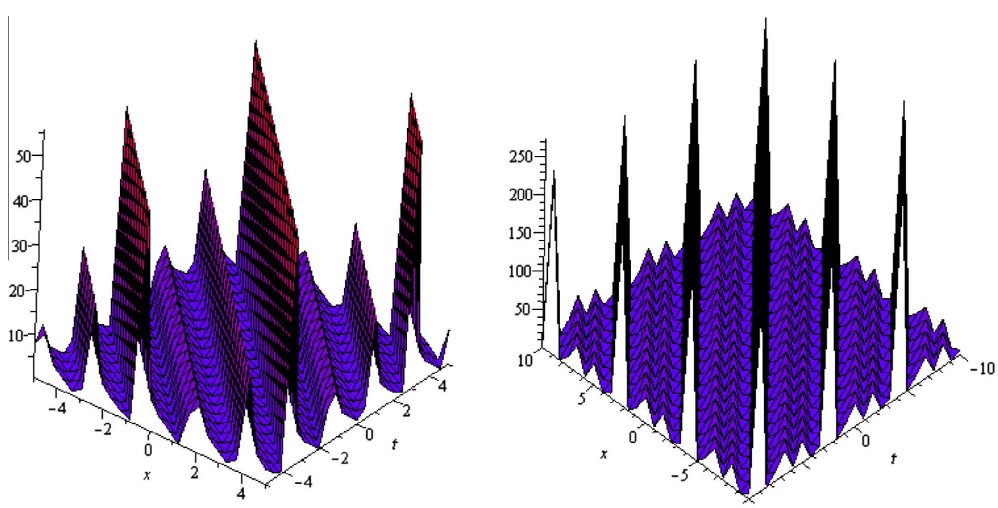

Figure 2 3D plot of the periodic traveling wave solution of u_3 and v_3 with $-10 \leqslant x,t \leqslant 10$ respectively.

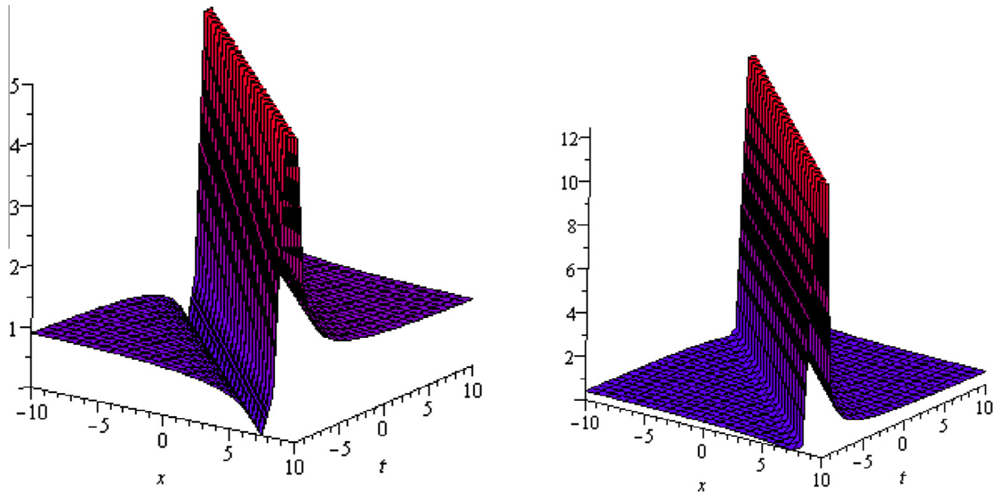

Figure 3 3D plot of the singular Kink traveling wave solution of u_9 and v_9 with $-10 \leqslant x,t \leqslant 10$ respectively.

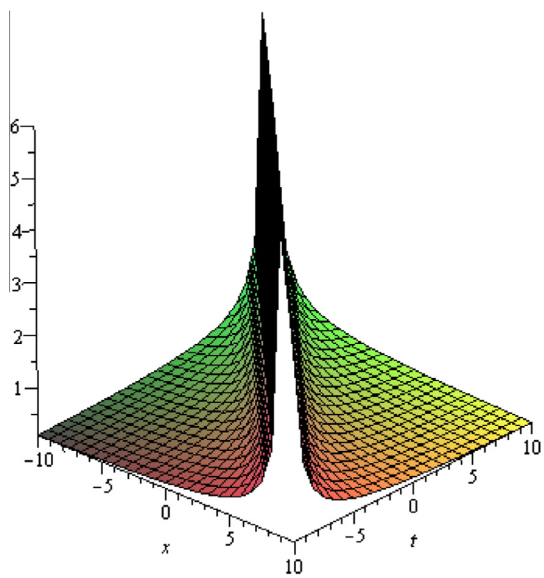

Figure 4 3D plot of cuspon solution of u_{20} with $-10 \leqslant x,t \leqslant 10$, $y = 0$.

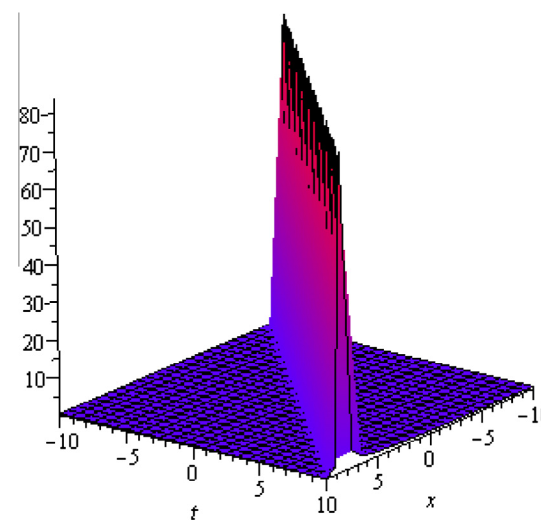

Figure 5 3D plot of singular cuspon solution of v_8 with $-10 \leqslant x,t \leqslant 10$.

Solutions $u_5, v_5, u_6, v_6, u_7, v_7, u_8, u_9, v_9, u_{10}$ and v_{10} of the coupled Higgs equation (11) represent the singular kink-type traveling wave solutions. Fig. 3 shows the shape of the exact singular kink-type solution of u_9 and v_9 for $r = 1$, $c = 1$, $\lambda = 0$, $\mu = 0$, $A_0 = 1$, $A_1 = 2$ and $E = 0.5$ with $-10 \leqslant x, t \leqslant 10$ respectively. We note that kink solution acquires non-vanishing values as $x \to \infty$. For convenience the other figures are omitted.

Solutions v_{15} of the coupled Higgs Eq. (11) and u_{19}, u_{20} of the Maccari system (35) for $y = 0$ are presented cuspon. Cuspons are another form of solitons where solution exhibits cusps at their crests. Fig. 4 below shows the cuspon of u_{20} for $p = 1$, $q = 2$, $\lambda = 0$, $\mu = 0$, $A_1 = 2$ and $E = 0.5$ with $-10 \leqslant x, t \leqslant 10, y = 0$.

Fig. 5 below shows the singular cuspon of the coupled Higgs equation, obtained from solution v_8 for $c = -1$, $\lambda = 1$, $\mu = 2$, $A_0 = 1$, $A_1 = 2$ and $E = 0.5$ with $-10 \leqslant x, t \leqslant 10$.

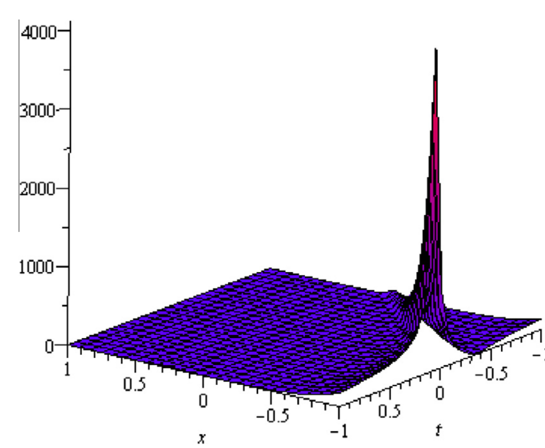

Figure 6 3D plot of solitary wave solution of u_{12} with $-10 \leqslant x,t \leqslant 10$.

Finally, solutions u_{12}, u_{13}, u_{14} and u_{15} of the coupled Higgs equation (11) represent the solitary wave solutions. Fig. 6 shows the shape of the exact solitary wave solutions of u_{12} for $r = 1$, $c = 1$, $\lambda = 2$, $\mu = 3$ and $E = 0.5$ with $-10 \leqslant x, t \leqslant 10$.

5. Comparisons

Many authors applied different methods to the coupled Higgs equation and the Maccari system for obtaining traveling wave solutions, for instance, Lee and Sakthivel (Lee and Sakthivel, 2013) used the Kudryashov method for finding traveling wave solutions to the coupled Higgs equation and the Maccari system. The obtaining solutions $v_5(x, t)$, $u_6(x, t)$, $v_6(x, t)$, $u_{13}(x, t)$, $v_{13}(x, t)$ in this article are equivalent to the solutions $u_1(x, t)$, $v_1(x, t)$, $u_2(x, t)$, $v_2(x, t)$, $u_3(x, t)$, $v_3(x, t)$ found in Lee and Sakthivel (2013) for the Higgs field equations. The solutions $u_{16}(x, y, t)$, $v_{13}(x, y, t)$ found in Lee and Sakthivel (2013) are equivalent to our obtained solutions $u_{16}(x, y, t)$, $v_{16}(x, y, t)$ for the Maccari system. If we set $k_2 = 0$, $n = 1$ and $q_{yy} = 0$, then the generalized Maccari system (see Ref. Ahmed et al., 2013) can be reduced to the Maccari system (35). Ahmed et al. (2013) applied the mapping method and Lie symmetry analysis to obtain solitons and other solutions of the generalized Maccari system. They studied the solitons by combining the linear and power low nonlinearity effects. In this article, our obtained solutions u_{16}, v_{16}, u_{18}, v_{18}, v_{19} also represent solitary wave solutions and solitons are originated according to the variation of the physical parameters. Therefore, if we take the particular values of the physical parameters, some of our obtained solutions coincide with some of the particular solutions obtained by other methods mentioned in the references (Lee and Sakthivel, 2013; Bahrami et al., 2011; Manafian and Zamanpour, 2013a,b). By means of this scheme, we found some new traveling wave solutions of the above mentioned equations. Therefore, the $\exp(-\varphi(\xi))$-expansion method provides some new exact solutions which are not found in other literature. This is the main advantage of this method. The graphical representation describes the behaviors of the oscillatory particle in the Higgs field or any field which satisfies the coupled Higgs equation and the Maccari system. The solutions obtained in this article have been checked by putting them back into the original equation and found correct.

6. Conclusions

In this article, we considered two complex coupled equations and the $\exp(-\Phi(\xi))$-expansion method has been successfully implemented to obtain new generalized traveling wave solutions of the coupled Higgs equation and the Maccari system. The obtained solutions are expressed by the hyperbolic functions, trigonometric functions and rational functions. These types of solutions have many potential applications in nonlinear optics, fluid mechanics, quantum field theory, complex scalar nucleon field, and plasma physics. It is important to point out that the $\exp(-\Phi(\xi))-$expansion method is direct, concise, elementary and comparing to other methods, like the tanh-coth method, Jacobi elliptic function method, Exp-function method it is easier and effective for obtaining exact solutions for a wide class of coupled nonlinear problems. We have applied this method only in two coupled nonlinear equations, but it can be further applied to other coupled equations to establish new reliable solutions. The graphical representations explicitly reveal the high applicability and competence of the proposed algorithm.

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