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Original article

New solitary wave structures to the (2 + 1)-dimensional KD and KP equations with spatio-temporal dispersion

Md Nur Alam [a,b], Cemil Tunç [c,*]

[a] School of Mathematical Sciences, University of Science and Technology of China, Hefei 230026 China
[b] Department of Mathematics, Pabna University of Science and Technology, Pabna 6600, Bangladesh
[c] Department of Mathematics, Faculty of Sciences, Van Yuzuncu Yil University, 65080 Van, Turkey

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ABSTRACT

The present paper studies the novel generalized (G'/G)-expansion technique to two nonlinear evolution equations: The $(2+1)$-dimensional Konopelchenko-Dubrovsky (KD) equation and the $(2+1)$-dimensional Kadomtsev-Petviashvili (KP) equation and acquires some new exact answers. The secured answers include a particular variety of solitary wave solutions, such as periodic, compaction, cuspon, kink, soliton, a bright periodic wave, Bell shape soliton, dark periodic wave and various kinds of soliton of the studied equation are achieved. These new particular kinds of solitary wave solutions will improve the earlier solutions and help us understand the physical meaning further and interpret the nonlinear generation of nonlinear wave equations of fluid in an elastic tube and liquid, including small bubbles and turbulence and the acoustic dust waves in dusty plasmas. Additionally, the studied approach could also be employed to obtain exact wave solutions for the other nonlinear evolution equations in applied sciences.

1. Introduction

Non-linear evolution equations perform a critical task to models of many applied mathematics, mathematical physics and engineering, for example, optical fibres, fluid mechanics, plasma waves, complex scalar nucleon field, non-linear optics, quantum field theory, fibre optics, solid-state physics, chemical physics, plasma physics, ocean engineering etc. The exact wave answers of non-linear wave models are executed to describe the physical device in real life. For this goal, the distinct information of the exact answer of non-linear evolution equations is hot subjects in advanced research areas. There are numerous reliable, and robust procedures have been improved to examine exact wave solutions of non-linear evolution equations, for example, improved $exp(-\phi(\xi))$-expansion method (Chen et al., 2019), tanh-coth expansion method (Alquran et al., 2018), the exponential rational function method (Tebue et al., 2016), $exp(-\phi(\xi))$-expansion method (Alam and Belgacem, 2016; Alam and Tunc, 2016), the Power Index Method (Shrauner, 2019), Bernoulli sub-equation method (Syam, 2019), Bcklund transformation method (Liu et al., 2019), Weierstrass elliptic function method (Krishnan and Peng, 2005), Lie symmetry approach (Ren et al., 2019), Lie point symmetries (Khalique and Moleleki, 2019), Darboux transformation method (Chen et al., 2018), singular manifold method (Peng and Krishnan, 2005), N-fold Darboux transformation (Ha et al., 2019), the simplified Hirotas method (Wazwaz and El-Tantawy, 2017), the novel (G'/G)-expansion technique (Alam et al., 2014a; Alam and Akbar, 2014; Akbar et al., 2016), the extended modified $exp(-\phi(\xi))$ function method (Karaagac et al., 2019),the homotopy perturbation method (Shqair, 2019), the sine-Gordon expansion method (Bulut et al., 2017; Bulut et al., 2018), the improved Bernoulli sub-equation function method (Baskonus and Bulut, 2016), rational sine-cosine method (Alqurana et al., 2019) and so on.

* Corresponding author.
E-mail addresses: nuralam23@mail.ustc.edu.cn (M.N. Alam), cemtunc@yahoo.com (C. Tunç).
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Firstly, we are consider the $(2+1)$-dimensional KD equation (Konopelchenko and Dubrovsky, 1984; Song and Zhang, 2006; Wazwaz, 2007a; Wazwaz, 2007b; Taghizadeh and Mirzazadeh, 2011) as

$$u_t - u_{xxx} - 6buu_x + \frac{2}{3}a^2u^2u_x - 3v_y + 3au_xv = 0, \tag{1}$$

$$u_y = v_x, \tag{2}$$

where a and b are real parameters. Eqs. (1) and (2) including two spatial and one temporal dimensions.

Finally, we are consider the $(2+1)$-dimensional KP equation (Kadomtsev and Petviashvili, 1970) of the form

$$(u_t + 6uu_x + u_{xxx})_x + \lambda u_{yy} = 0, \tag{3}$$

where λ is a real free unknown constant and $u = u(x,y,t)$ is a function of x, y and t. It provides an analysis of the general weakly dispersive waves and is also implemented to form shallow-water waves including weakly non-linear restoring forces. Numerous procedures for attaining exact wave answers of the studied equation have been recommended, for example, Hirota's method (Hereman and Nuseir, 1997), sine-cosine algorithm (Wazwaz, 2004), the extended mapping method (Peng and Krishnan, 2005), the Hirotas bilinear method (Wazwaz, 2012), etc. In this article, we produce the novel generalized $(\frac{G'}{G})$-expansion system to discover some exact and novel solutions for the considered models. And also received the special kind of solitary wave solutions such as compaction, cuspon, kink, periodic, soliton traveling wave solution, singular soliton traveling wave solution and various kinds of solutions of the studied equation through the novel generalized $(\frac{G'}{G})$-expansion method (Alam et al., 2014b; Alam, 2015; Alam and Li, 2019). Accordingly, the paper is designed of the forms. For some other the related papers, we refer the readers to (Alam and Tunç, 2020a; Alam and Tunç, 2020b; Alam and Tunç, 2020c; Alam et al., 2020). Section 2 shows the novel generalized (G'/G)-expansion method. And the exact and new solutions of the $(2+1)$-dimensional potential KP and KD equations are expressed applying the studied method in Section 3. Finally, in Section 4, conclusion is described.

2. Methodology

This section instantly highlights the significant ideas of the novel generalized $(\frac{G'}{G})$-expansion approach:

- **Step 1:** We assume that a nonlinear evolution equation including x, y and t as follows:

$$\mathrm{P}(u, u_t, u_x, u_y, u_{yy}, u_{tt}, u_{xx}, u_{xt}, \ldots) = 0, \tag{4}$$

where $u(x,y,t)$ is the result of Eq. (4).

- **Step 2:** Take the solutions of Eq. (4) as follows:

$$\mathrm{u}(x,t) = u(\xi), \xi = x + y + Vt, \tag{5}$$

where V is the transformation variable and under the transformation Eq. (5), Eq. (4) converts a nonlinear ODE:

$$R(u, \frac{\partial u}{\partial \xi}, \frac{\partial^2 u}{\partial \xi^2}, \ldots) = 0. \tag{6}$$

- **Step 3:** Calculate N by using balance rule in Eq. (6).
- **Step 4:** Consider that Eq. (6) has the solution can be represent as

$$u(\xi) = a_0 + a_1(d+H) + \frac{b_1}{d+H} + a_2(d+H)^2 + \frac{b_2}{(d+H)^2} + \sum_{i=3}^{N}(a_i(d+H)^i + b_i(d+H^{-i}), \tag{7}$$

where a_0, a_1, a_2, b_1, b_2 and d are constants. $H(\xi) = (\frac{G'}{G})$ and $G = G(\xi)$ represent the equation

$$AGG'' - BGG' - EG^2 - C(G')^2 = 0, \tag{8}$$

where A, B, C and E are real unknown free parameters. Eq. (8) has three solutions:

– When $B \neq 0, \Psi = A - C$ and $\Omega = B^2 + 4E(A-C) > 0$, the solution of $H(\xi) = (\frac{G'}{G})$ is

$$H(\xi) = \frac{G'}{G} = \frac{B}{2\Psi} + \frac{\sqrt{\Omega}}{2\Psi}\frac{C_1 sinh(\frac{\sqrt{\Omega}\xi}{2A}) + C_2 cosh(\frac{\sqrt{\Omega}\xi}{2A})}{C_1 cosh(\frac{\sqrt{\Omega}\xi}{2A}) + C_2 sinh(\frac{\sqrt{\Omega}\xi}{2A})}; \tag{9}$$

– When $B \neq 0, \Psi = A - C$ and $\Omega = B^2 + 4E(A-C) < 0$, the solution of $H(\xi) = (\frac{G'}{G})$ is

$$H(\xi) = \frac{G'}{G} = \frac{B}{2\Psi} + \frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1 sin(\frac{\sqrt{-\Omega}\xi}{2A}) + C_2 cos(\frac{\sqrt{-\Omega}\xi}{2A})}{C_1 cos(\frac{\sqrt{-\Omega}\xi}{2A}) + C_2 sin(\frac{\sqrt{-\Omega}\xi}{2A})}; \tag{10}$$

– When $B \neq 0, \Psi = A - C$ and $\Omega = B^2 + 4E(A-C) = 0$, the solution of $H(\xi) = (\frac{G'}{G})$ is

$$H(\xi) = \frac{G'}{G} = \frac{B}{2\Psi} + \frac{C_2}{C_1 + C_2\xi}. \tag{11}$$

- **Step 5:** Calculate the coefficients of $(d+H)^N$ and $(d+H)^{-N}$ and receive a set of algebraic equations for a_i, b_i, d and V by inserting each coefficient to zero. Receive the exact wave solutions of Eq. (4) through setting the received values in Eqs. (7) and (6) including the amount of N.

3. The (2 + 1)-dimensional Konopelchenko-Dubrovsky (KD) equation

In this section, the method is utilised to obtain soliton for the (2 + 1)-dimensional KD equation. We consider that the (2 + 1)-dimensional KD equation:

$$u_t - u_{xxx} - 6buu_x + \frac{2}{3}a^2u^2u_x - 3v_y + 3au_xv = 0, \tag{12}$$

$$u_y = v_x, \tag{13}$$

where a and b are real parameters.

Using the wave variable $\xi = x + y - ct$ carries the KD Eqs. (12) and (13) into a system of ODE:

$$-cu' - u''' - 3b(u^2)' + 0.5a^2(u^3)' - 3v' + 3au'v = 0. \tag{14}$$

$$u' = v'. \tag{15}$$

Integrating on the Eq. (15), we have

$$u = v. \tag{16}$$

From Eqs. (16) and (14), we obtain

$$(c+3)u + 3(b - 0.5a)u^2 - 0.5a^2u^3 + u'' = 0, a \neq 0. \tag{17}$$

If $a = 2b$, then from Eq. (17), we obtain

$$(c+3)u - 0.5a^2u^3 + u'' = 0, a \neq 0. \tag{18}$$

According to the novel generalized $(\frac{G'}{G})$-expansion scheme (Alam et al., 2014a; Alam, 2015; Alam and Li, 2019), applying homogeneous balance rule between u'' with u^3 gives $N = 1$. Therefore, the Eq. (18) has the following solution:

$$\mathrm{u}(\xi) = a_0 + a_1(d+H) + b_1(d+H)^{-1}, \tag{19}$$

where a_0, a_1, b_1 and d are constants. Using Eq. (19) and (8) into Eq. (18), the left-hand side of Eq. (18) is converts into a polynomial of

$(d+H)^N$ and $(d+H)^{-N}$. Plugging Maple, we determine the system converted into received algebraic equations as the following:

- The first set:

$$c = -\frac{6A^2-4E\Phi-B^2}{2A^2},\ a_0 = \frac{2d\Phi+B}{Aa},\ \Phi = A-C,$$
$$a_1 = -\frac{2\Phi}{Aa},\ b_1 = 0.$$

- The second set:

$$c = -\frac{12A^2-4E\Phi-B^2}{4A^2},\ d = -\frac{B}{2\Phi},\ a_0 = 0,\ a_1 = 0,$$
$$b_1 = \frac{\pm\sqrt{0.5}(4E\Phi+B^2)}{2\Phi aA}.$$

- The third set:

$$c = -\frac{6A^2-4E\Phi-B^2}{2A^2},\ a_0 = -\frac{2d\Phi+B}{Aa},\ a_1 = \frac{2\Phi}{Aa},\ b_1 = 0.$$

Using the values of the first set into Eq. (19) into Eq. (18), we have the exact solutions as follows:

$$u_1(x,y,t) = \left\{\frac{2d\Phi+B}{Aa} - \frac{2\Phi}{Aa}(d+\frac{B}{2\Phi}+\frac{\sqrt{\Omega}}{2\Phi}tanh(\frac{\sqrt{\Omega}}{2A})(x+y+\frac{6A^2-4E\Phi-B^2}{2A^2}t))\right\}.$$

$$u_2(x,y,t) = \left\{\frac{2d\Phi+B}{Aa} - \frac{2\Phi}{Aa}(d+\frac{B}{2\Phi}+\frac{\sqrt{\Omega}}{2\Phi}coth(\frac{\sqrt{\Omega}}{2A})(x+y+\frac{6A^2-4E\Phi-B^2}{2A^2}t))\right\}.$$

$$u_3(x,y,t) = \left\{\frac{2d\Phi+B}{Aa} - \frac{2\Phi}{Aa}(d+\frac{B}{2\Phi}-\frac{\sqrt{-\Omega}}{2\Phi}tan(\frac{\sqrt{-\Omega}}{2A})(x+y+\frac{6A^2-4E\Phi-B^2}{2A^2}t))\right\}.$$

$$u_4(x,y,t) = \left\{\frac{2d\Phi+B}{Aa} - \frac{2\Phi}{Aa}(d+\frac{B}{2\Phi}+\frac{\sqrt{-\Omega}}{2\Phi}cot(\frac{\sqrt{-\Omega}}{2A})(x+y+\frac{6A^2-4E\Phi-B^2}{2A^2}t))\right\}.$$

$$u_5(x,y,t) = \left\{\frac{2d\Phi+B}{Aa} - \frac{2\Phi}{Aa}(d+\frac{B}{2\Phi}+\frac{C_2}{C_1+C_2(x+y+\frac{6A^2-4E\Phi-B^2}{2A^2}t)}\right\}.$$

$$u_6(x,y,t) = \left\{\frac{2d\Phi+B}{Aa} - \frac{2\Phi}{Aa}(d+\frac{\sqrt{\Delta}}{\Phi}tanh(\frac{\sqrt{\Delta}}{A})(x+y+\frac{6A^2-4E\Phi-B^2}{2A^2}t))\right\}.$$

$$u_7(x,y,t) = \left\{\frac{2d\Phi+B}{Aa} - \frac{2\Phi}{Aa}(d+\frac{\sqrt{\Delta}}{\Phi}coth(\frac{\sqrt{\Delta}}{A})(x+y+\frac{6A^2-4E\Phi-B^2}{2A^2}t))\right\}.$$

$$u_8(x,y,t) = \left\{\frac{2d\Phi+B}{Aa} - \frac{2\Phi}{Aa}(d-\frac{\sqrt{-\Delta}}{\Phi}tan(\frac{\sqrt{-\Delta}}{A})(x+y+\frac{6A^2-4E\Phi-B^2}{2A^2}t))\right\}.$$

$$u_9(x,y,t) = \left\{\frac{2d\Phi+B}{Aa} - \frac{2\Phi}{Aa}(d+\frac{\sqrt{-\Delta}}{\Phi}cot(\frac{\sqrt{-\Delta}}{A})(x+y+\frac{6A^2-4E\Phi-B^2}{2A^2}t))\right\}.$$

Figs. 1–5 manifest two of the answers under $2D, 3D$ and the contour plot surfaces by applying Maple.

4. The (2 + 1)-dimensional Kadomtsev-Petviashvili equation

In this section, the method is utilised to obtain compaction, cuspon, kink, periodic, soliton traveling wave solution, singular soliton traveling wave solution and various kinds of solutions for the $(2+1)$-dimensional KP equation which are fundamental nonlinear evolution equations in the field of nonlinear dynamics. We consider the $(2+1)$-dimensional KP equation:

$$(u_t + 6uu_x + u_{xxx})_x + \lambda u_{yy} = 0. \tag{20}$$

Applying $u(\xi) = v(x,y,t), \xi = x+y-Vt$ provides the Eq. (20) into a nonlinear ODE:

$$(\lambda - V)u + 3u^2 + u'' = 0. \tag{21}$$

It is taken upon integrating twice and putting coefficients of integration to zeros. Making the homogeneous rule in Eq. (21), we obtain $N = 2$ and the solution as follows:

$$u(\xi) = a_0 + a_1(d+H) + a_2(d+H)^2 + b_1(d+H)^{-1} + b_2(d+H)^{-2}, \tag{22}$$

where a_0, a_1, a_2, b_1, b_2 and d are constants. Plugging (22) and (8) into (21), the left-hand side of Eq. (21) is changed into $(d+H)^N$ and $(d+H)^{-N}$. Plugging Maple, we determine the system of received algebraic equations as the following:

- The first set: $a_0 = \frac{B^2+4E\Psi}{A^2},\ a_1 = 0,\ a_2 = -\frac{2\Psi^2}{A^2},\ d = -\frac{B}{2\Psi},\ b_1 = 0,\ b_2 = -\frac{m_1}{8A^2\Psi^2},\ V = \frac{\lambda A^2+16E\Psi+4B^2}{A^2}$, where $\Psi = A-C, m_1 = B^4+8EB^2\Psi+16E^2\Psi^2, A, B, C$ and E are free constants.
- The second set: $a_0 = -\frac{2(-E\Psi+Bd\Psi+d^2\Psi^2)}{A^2},\ a_1 = \frac{2(B\Psi+2d\Psi^2)}{A^2},\ a_2 = -\frac{2\Psi^2}{A^2},\ d = d,\ b_1 = 0,\ b_2 = 0,\ V = \frac{\lambda A^2+4E\Psi+4B^2}{A^2}$, where $\Psi = A-C, A, B, C$ and E are free constants.
- The third set: $a_0 = -\frac{B^2-2E\Psi+6Bd\Psi+6d^2\Psi^2}{3A^2},\ a_1 = \frac{2(B\Psi+2d\Psi^2)}{A^2},\ a_2 = -\frac{2\Psi^2}{A^2},\ d = d,\ b_1 = 0,\ b_2 = 0,\ V = \frac{\lambda A^2-4E\Psi+4B^2}{A^2}$, where $\Psi = A-C, A, B, C$ and E are free constants.
- The fourth set: $a_0 = \frac{B^2+4E\Psi}{A^2},\ a_1 = 0,\ a_2 = 0,\ d = -\frac{B}{2\Psi},\ b_1 = 0,\ b_2 = -\frac{m_2}{8A^2\Psi^2},\ V = \frac{\lambda A^2+4E\Psi+4B^2}{A^2}$, where $\Psi = A-C, m_2 = (B^2+E\Psi)^2, A, B, C$ and E are free constants.
- The fifth set: $a_0 = \frac{B^2+4E\Psi}{6A^2},\ a_1 = 0,\ a_2 = 0,\ d = -\frac{B}{2\Psi},\ b_1 = 0,\ b_2 = -\frac{m_3}{8A^2\Psi^2},\ V = \frac{\lambda A^2-4E\Psi-B^2}{A^2}$, where $\Psi = A-C, m_3 = (B^2+E\Psi)^2, A, B, C$ and E are free constants.
- The sixth set: $a_0 = \frac{2m_4}{A^2},\ a_1 = 0,\ a_2 = 0,\ d = d,\ b_1 = -\frac{2m_5}{A^2},\ b_2 = -\frac{2m_6}{A^2},\ V = \frac{\lambda A^2+4E\Psi+B^2}{A^2}$, where $\Psi = A-C, m_4 = -(-E\Psi+8d\Psi+d^2\Psi^2), m_5 = (2d^3\Psi^2-2Ed\Psi-EB+B^2d+3Bd^2\Psi)$, $m_6 = -(d^4\Psi^2-2Ed^2\Psi+2Bd^3\Psi+B^2d^2-2BdE+E^2), A, B, C$ and E are free constants.
- The seventh set: $a_0 = \frac{2m_7}{3A^2},\ a_1 = 0,\ a_2 = 0,\ d = d,\ b_1 = -\frac{2m_8}{A^2},\ b_2 = -\frac{2m_9}{A^2},\ V = \frac{\lambda A^2-4E\Psi-B^2}{A^2}$, where $\Psi = A-C, m_7 = -(-B^2-2E\Psi+6Bd\Psi+6d^2\Psi^2), m_8 = (2d^3\Psi^2-2Ed\Psi-EB+B^2d+3Bd^2\Psi)$, $m_9 = -(d^4\Psi^2-2Ed^2\Psi+2Bd^3\Psi+B^2d^2-2BdE+E^2)$, A, B, C and E are free constants.

By putting the values of the first set including Eq. (8) into Eq. (22) and simplifying, leads to the following exact wave solutions:

$$u_{11}(\xi) = \frac{1}{2A^2}(2(B^2+4E\Psi) - \Omega coth^2(\frac{\sqrt{\Omega}\xi}{2A}) - \frac{m_1}{\Omega}tanh^2(\frac{\sqrt{\Omega}\xi}{2A})),$$

$$u_{12}(\xi) = \frac{1}{2A^2}(2(B^2+4E\Psi) - \Omega tanh^2(\frac{\sqrt{\Omega}\xi}{2A}) - \frac{m_1}{\Omega}coth^2(\frac{\sqrt{\Omega}\xi}{2A})),$$

$$u_{13}(\xi) = \frac{1}{2A^2}(2(B^2+4E\Psi) + \Omega cot^2(\frac{\sqrt{-\Omega}\xi}{2A}) + \frac{m_1}{\Omega}tan^2(\frac{\sqrt{-\Omega}\xi}{2A})),$$

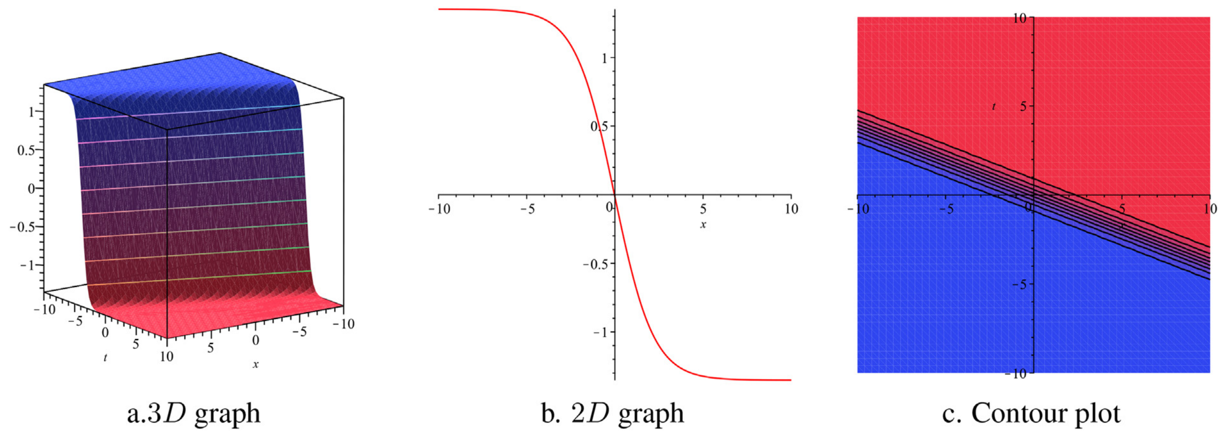


Fig. 1. Graphical representation of $u_1(x,y,t)$ for $y=0, A=4, B=1, C=1, d=1, a=0.5$ and $E=1$ within the interval $-1 \leqslant x,t \leqslant 1$ and $t=0.01$ for two dimensional.

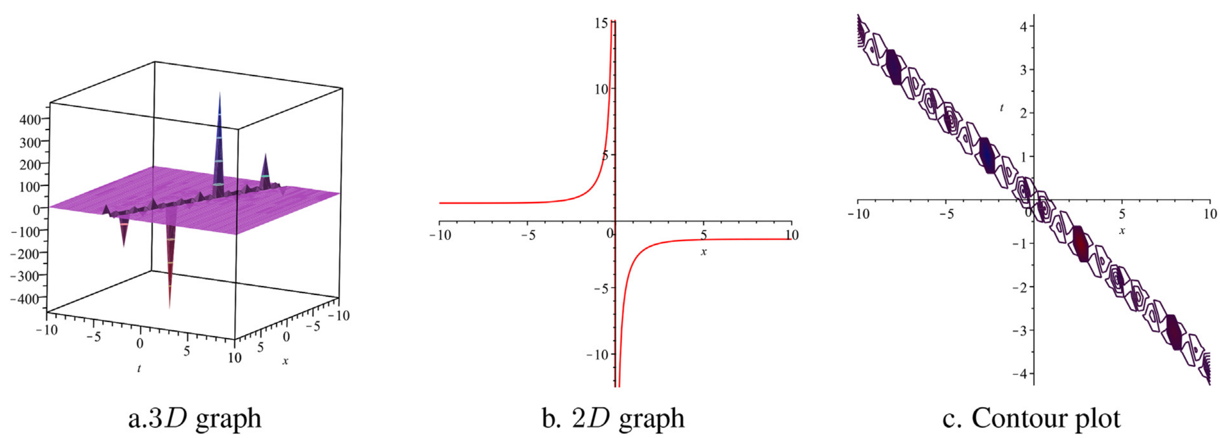


Fig. 2. Graphical representation of $u_2(x,y,t)$ for $y=0, A=4, B=1, C=1, d=1, a=0.5$ and $E=1$ within the interval $-1 \leqslant x,t \leqslant 1$ and $t=0.01$ for two dimensional.

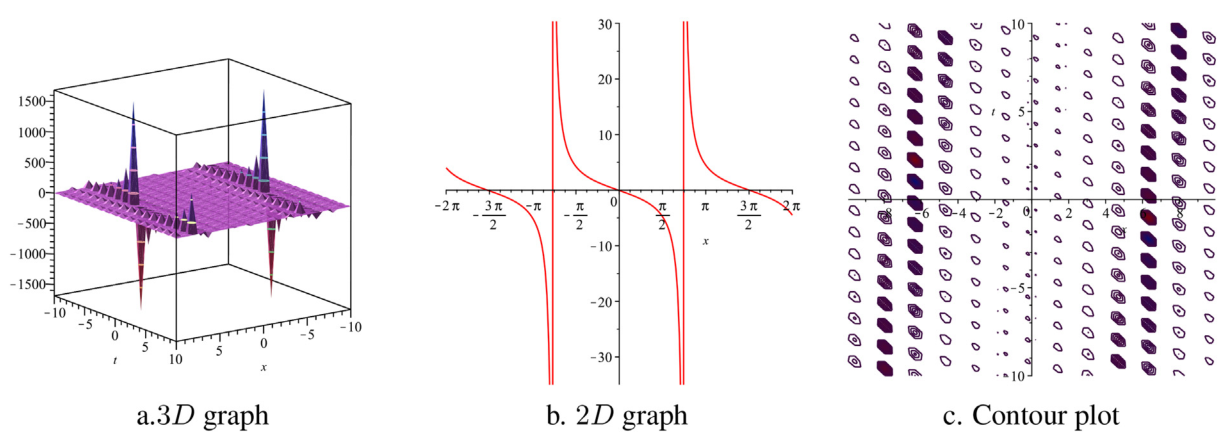
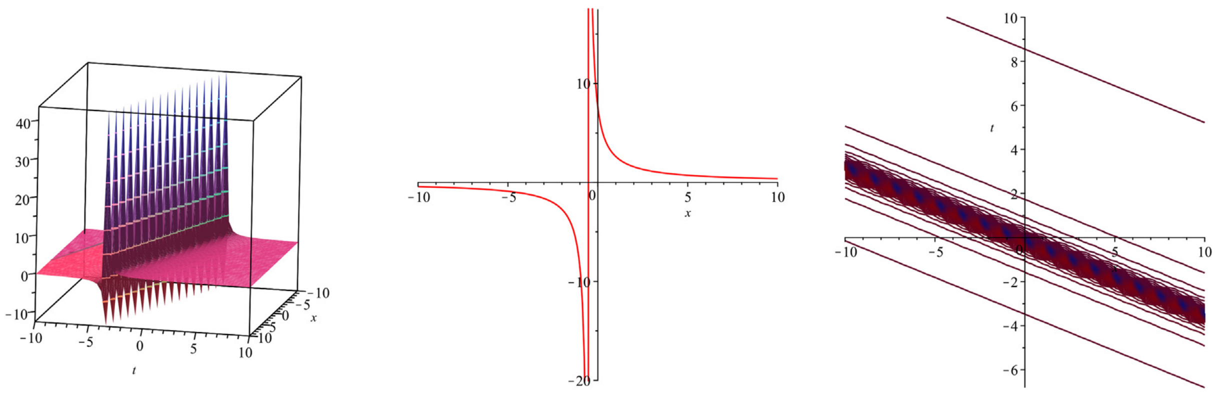


Fig. 3. Graphical representation of $u_3(x,y,t)$ for $y=0, A=2, B=1, C=4, d=1, a=0.5$ and $E=1$ within the interval $-1 \leqslant x,t \leqslant 1$ and $t=0.01$ for two dimensional.

$$u_{14}(\xi) = \frac{1}{2A^2}(2(B^2+4E\Psi) + \Omega tan^2(\frac{\sqrt{-\Omega}\xi}{2A}) + \frac{m_1}{\Omega}cot^2(\frac{\sqrt{-\Omega}\xi}{2A})),$$

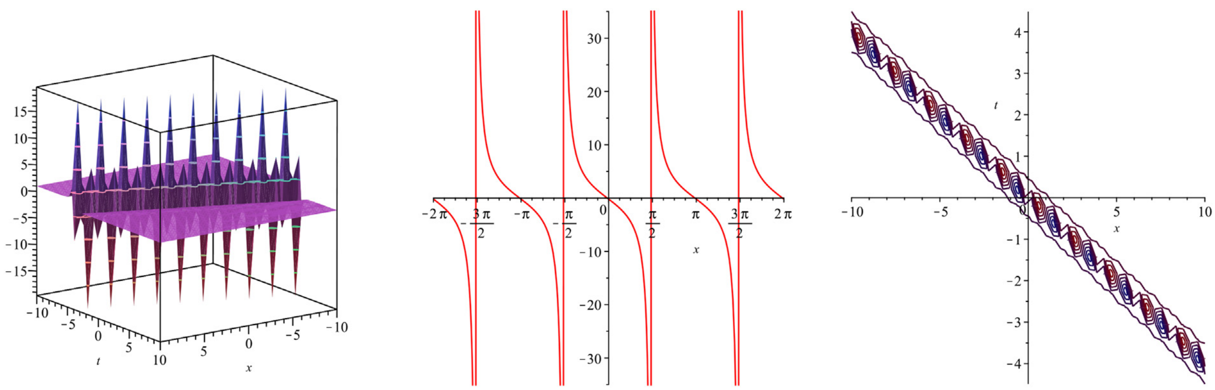
$$u_{15}(\xi) = \frac{1}{A^2}((B^2+4E\Psi) - \frac{2\Psi^2}{A^2}(\frac{C_2}{C_1+C_2\xi})^2 - \frac{m_1}{8A^2\Psi^2}(\frac{C_2}{C_1+C_2\xi})^{-2}),$$

$$u_{16}(\xi) = \frac{1}{A^2}((B^2+4E\Psi) - \frac{2\Psi^2}{A^2}(\frac{-B}{2\Psi} + \frac{\sqrt{-\Delta}}{\Psi}coth(\frac{\sqrt{-\Delta}\xi}{A}))^2 - \frac{m_1}{8A^2\Psi^2}(\frac{-B}{2\Psi} + \frac{\sqrt{-\Delta}}{\Psi}coth(\frac{\sqrt{-\Delta}\xi}{A}))^{-2}),$$



a.3D graph b. 2D graph c. Contour plot

Fig. 4. Graphical representation of $u_5(x,y,t)$ for $y=0, A=1, B=1, C=2, d=1, a=0.5$ and $E=1$ within the interval $-1\leqslant x,t\leqslant 1$ and $t=0.01$ for two dimensional.



a.3D graph b. 2D graph c. Contour plot

Fig. 5. Graphical representation of $u_7(x,y,t)$ for $y=0, A=4, B=0, C=1, d=1, a=0.5$ and $E=5$ within the interval $-1\leqslant x,t\leqslant 1$ and $t=0.01$ for two dimensional.

$$u_{17}(\xi)=\frac{1}{A^2}((B^2+4E\Psi)-\frac{2\Psi^2}{A^2}(\frac{-B}{2\Psi}+\frac{\sqrt{-\Delta}}{\Psi}tanh(\frac{\sqrt{-\Delta}\xi}{A}))^2-\frac{m_1}{8A^2\Psi^2}(\frac{-B}{2\Psi}+\frac{\sqrt{-\Delta}}{\Psi}tanh(\frac{\sqrt{-\Delta}\xi}{A}))^{-2}),$$

$$u_{18}(\xi)=\frac{1}{A^2}((B^2+4E\Psi)-\frac{2\Psi^2}{A^2}(\frac{-B}{2\Psi}+\frac{\sqrt{-\Delta}}{\Psi}cot(\frac{\sqrt{-\Delta}\xi}{A}))^2-\frac{m_1}{8A^2\Psi^2}(\frac{-B}{2\Psi}+\frac{\sqrt{-\Delta}}{\Psi}cot(\frac{\sqrt{-\Delta}\xi}{A}))^{-2}),$$

$$u_{19}(\xi)=\frac{1}{A^2}((B^2+4E\Psi)-\frac{2\Psi^2}{A^2}(\frac{-B}{2\Psi}+\frac{\sqrt{\Delta}}{\Psi}tan(\frac{\sqrt{-\Delta}\xi}{A}))^2-\frac{m_1}{8A^2\Psi^2}(\frac{-B}{2\Psi}+\frac{\sqrt{\Delta}}{\Psi}tan(\frac{\sqrt{-\Delta}\xi}{A}))^{-2}),$$

where $\xi=x+y-(\frac{\lambda A^2+16E\Psi+4B^2}{A^2})t$.

Similarly for the second set, we have:

$$u_{21}(\xi)=\frac{1}{2A^2}((B^2+4E\Psi)-\Omega coth^2(\frac{\sqrt{\Omega}\xi}{2A})),$$

$$u_{22}(\xi)=\frac{1}{2A^2}((B^2+4E\Psi)-\Omega tanh^2(\frac{\sqrt{\Omega}\xi}{2A})),$$

$$u_{23}(\xi)=\frac{1}{2A^2}((B^2+4E\Psi)+\Omega cot^2(\frac{\sqrt{-\Omega}\xi}{2A})),$$

$$u_{24}(\xi)=\frac{1}{2A^2}((B^2+4E\Psi)+\Omega tan^2(\frac{\sqrt{-\Omega}\xi}{2A})),$$

$$u_{25}(\xi)=\frac{1}{2A^2}((B^2+4E\Psi)-(\frac{2\Psi C_2}{C_1+C_2\xi})^2),$$

$$u_{26}(\xi)=\frac{2}{A^2}(E\Psi+\sqrt{\Delta}(Bcoth(\frac{\sqrt{\Delta}\xi}{A})-\sqrt{\Delta}coth^2(\frac{\sqrt{\Delta}\xi}{A}))),$$

$$u_{27}(\xi)=\frac{2}{A^2}(E\Psi+\sqrt{\Delta}(Btanh(\frac{\sqrt{\Delta}\xi}{A})-\sqrt{\Delta}tanh^2(\frac{\sqrt{\Delta}\xi}{A}))),$$

$$u_{28}(\xi)=\frac{2}{A^2}(E\Psi+\sqrt{\Delta}(IBcot(\frac{\sqrt{-\Delta}\xi}{A})-\sqrt{-\Delta}cot^2(\frac{\sqrt{-\Delta}\xi}{A}))),$$

$$u_{29}(\xi)=\frac{2}{A^2}(E\Psi-\sqrt{\Delta}(IBtan(\frac{\sqrt{-\Delta}\xi}{A})-\sqrt{-\Delta}tan^2(\frac{\sqrt{-\Delta}\xi}{A}))),$$

where $\xi=x+y-(\frac{\lambda A^2+4E\Psi+4B^2}{A^2})t$.

Similarly again for the third set, we find:

$$u_{31}(\xi)=\frac{1}{6A^2}((B^2+4E\Psi)-3\Omega coth^2(\frac{\sqrt{\Omega}\xi}{2A})),$$

$$u_{32}(\xi)=\frac{1}{6A^2}((B^2+4E\Psi)-3\Omega tanh^2(\frac{\sqrt{\Omega}\xi}{2A})),$$

$$u_{33}(\xi)=\frac{1}{6A^2}((B^2+4E\Psi)+3\Omega cot^2(\frac{\sqrt{-\Omega}\xi}{2A})),$$

$$u_{34}(\xi)=\frac{1}{6A^2}((B^2+4E\Psi)+3\Omega tan^2(\frac{\sqrt{-\Omega}\xi}{2A})),$$

$$u_{35}(\xi)=\frac{1}{6A^2}((B^2+4E\Psi)-3(\frac{2\Psi C_2}{C_1+C_2\xi})^2),$$

$$u_{36}(\xi)=\frac{2}{3A^2}((-B^2+2E\Psi)+6\sqrt{\Delta}(Bcoth(\frac{\sqrt{\Delta}\xi}{A}-\sqrt{\Delta}coth^2(\frac{\sqrt{\Delta}\xi}{A}))),$$

$$u_{37}(\xi)=\frac{2}{3A^2}((-B^2+2E\Psi)+6\sqrt{\Delta}(Btanh(\frac{\sqrt{\Delta}\xi}{A}-\sqrt{\Delta}tanh^2(\frac{\sqrt{\Delta}\xi}{A}))),$$

$$u_{38}(\xi)=\frac{1}{3A^2}((-B^2+2E\Psi)+6\sqrt{\Delta}(IBcot(\frac{\sqrt{-\Delta}\xi}{A}$$
$$-\sqrt{\Delta}cot^2(\frac{\sqrt{-\Delta}\xi}{A}))),$$

$$u_{39}(\xi)=\frac{2}{3A^2}((-B^2+2E\Psi)+6\sqrt{\Delta}(Btan(\frac{\sqrt{-\Delta}\xi}{A}+\sqrt{\Delta}tan^2(\frac{\sqrt{-\Delta}\xi}{A}))),$$

where $\xi=x+y-(\frac{\lambda A^2-4E\Psi+4B^2}{A^2})t$.

Similarly for the fourth set, we get:

$$u_{41}(\xi)=\frac{1}{2A^2}((B^2+4E\Psi)-\frac{m_2}{\Omega}coth^2(\frac{\sqrt{\Omega}\xi}{2A})),$$

$$u_{42}(\xi)=\frac{1}{2A^2}((B^2+4E\Psi)-\frac{m_2}{\Omega}tanh^2(\frac{\sqrt{\Omega}\xi}{2A})),$$

$$u_{43}(\xi)=\frac{1}{2A^2}((B^2+4E\Psi)+\frac{m_2}{\Omega}cot^2(\frac{\sqrt{-\Omega}\xi}{2A})),$$

$$u_{44}(\xi)=\frac{1}{2A^2}((B^2+4E\Psi)+\frac{m_2}{\Omega}tan^2(\frac{\sqrt{-\Omega}\xi}{2A})),$$

$$u_{45}(\xi)=\frac{1}{A^2}((B^2+4E\Psi)-\frac{m_2}{4\Psi^2}(\frac{C_2}{C_1+C_2\xi})^{-2}),$$

$$u_{46}(\xi)=\frac{1}{A^2}((B^2+4E\Psi)-\frac{m_2}{4\Psi^2}(\frac{-B}{2\Psi}+\frac{\sqrt{\Delta}}{\Psi}coth(\frac{\sqrt{\Delta}\xi}{A})^{-2}),$$

$$u_{47}(\xi)=\frac{1}{A^2}((B^2+4E\Psi)-\frac{m_2}{4\Psi^2}(\frac{-B}{2\Psi}+\frac{\sqrt{\Delta}}{\Psi}tanh(\frac{\sqrt{\Delta}\xi}{A})^{-2}),$$

$$u_{48}(\xi)=\frac{1}{A^2}((B^2+4E\Psi)-\frac{m_2}{4\Psi^2}(\frac{-B}{2\Psi}+\frac{\sqrt{-\Delta}}{\Psi}cot(\frac{\sqrt{-\Delta}\xi}{A})^{-2}),$$

$$u_{49}(\xi)=\frac{1}{A^2}((B^2+4E\Psi)-\frac{m_2}{4\Psi^2}(-\frac{-B}{2\Psi}-\frac{\sqrt{-\Delta}}{\Psi}tan(\frac{\sqrt{-\Delta}\xi}{A})^{-2}),$$

where $\xi=x+y-(\frac{\lambda A^2+4E\Psi+4B^2}{A^2})t$.

Similarly for the fifth set, we obtain:

$$u_{51}(\xi)=\frac{1}{6A^2}((B^2+4E\Psi)-\frac{3m_3}{\Omega}coth^2(\frac{\sqrt{\Omega}\xi}{2A})),$$

$$u_{52}(\xi)=\frac{1}{6A^2}((B^2+4E\Psi)-\frac{3m_3}{\Omega}tanh^2(\frac{\sqrt{\Omega}\xi}{2A})),$$

$$u_{53}(\xi)=\frac{1}{6A^2}((B^2+4E\Psi)+\frac{3m_3}{\Omega}cot^2(\frac{\sqrt{-\Omega}\xi}{2A})),$$

$$u_{54}(\xi)=\frac{1}{6A^2}((B^2+4E\Psi)+\frac{3m_3}{\Omega}tan^2(\frac{\sqrt{-\Omega}\xi}{2A})),$$

$$u_{55}(\xi)=\frac{1}{6A^2}((B^2+4E\Psi)-\frac{3m_3}{4\Psi^2}(\frac{C_2}{C_1+C_2\xi})^{-2}),$$

$$u_{56}(\xi)=\frac{1}{6A^2}((B^2+4E\Psi)-\frac{3m_3}{4\Psi^2}(\frac{-B}{2\Psi}+\frac{\sqrt{\Delta}}{\Psi}coth(\frac{\sqrt{\Delta}\xi}{A})^{-2}),$$

$$u_{57}(\xi)=\frac{1}{6A^2}((B^2+4E\Psi)-\frac{3m_3}{4\Psi^2}(\frac{-B}{2\Psi}+\frac{\sqrt{\Delta}}{\Psi}tanh(\frac{\sqrt{\Delta}\xi}{A})^{-2}),$$

$$u_{58}(\xi)=\frac{1}{6A^2}((B^2+4E\Psi)-\frac{3m_3}{4\Psi^2}(\frac{-B}{2\Psi}+\frac{\sqrt{-\Delta}}{\Psi}cot(\frac{\sqrt{-\Delta}\xi}{A})^{-2}),$$

$$u_{59}(\xi)=\frac{1}{A^2}((B^2+4E\Psi)-\frac{3m_3}{4\Psi^2}(-\frac{-B}{2\Psi}-\frac{\sqrt{-\Delta}}{\Psi}tan(\frac{\sqrt{-\Delta}\xi}{A})^{-2}),$$

where $\xi=x+y-(\frac{\lambda A^2-4E\Psi-B^2}{A^2})t$.

Similarly for the sixth set, we obtain:

$$\mathrm{u}_{61}(\xi)=\frac{2}{A^2}(m_4+m_5(d+\frac{B}{2\Omega}+\frac{\sqrt{\Omega}}{2\Psi}coth(\frac{\sqrt{\Omega}\xi}{2A})$$
$$+m_6(d+\frac{B}{2\Omega}+\frac{\sqrt{\Omega}}{2\Psi}coth(\frac{\sqrt{\Omega}\xi}{2A})^{-2}),$$

$$\mathrm{u}_{62}(\xi)=\frac{2}{A^2}(m_4+m_5(d+\frac{B}{2\Omega}+\frac{\sqrt{\Omega}}{2\Psi}tanh(\frac{\sqrt{\Omega}\xi}{2A})$$
$$+m_6(d+\frac{B}{2\Omega}+\frac{\sqrt{\Omega}}{2\Psi}tanh(\frac{\sqrt{\Omega}\xi}{2A})^{-2}),$$

$$\mathrm{u}_{63}(\xi)=\frac{2}{A^2}(m_4+m_5(d+\frac{B}{2\Omega}+\frac{\sqrt{-\Omega}}{2\Psi}cot(\frac{\sqrt{-\Omega}\xi}{2A})$$
$$+m_6(d+\frac{B}{2\Omega}+\frac{\sqrt{-\Omega}}{2\Psi}cot(\frac{\sqrt{-\Omega}\xi}{2A})^{-2}),$$

$$\mathrm{u}_{64}(\xi)=\frac{2}{A^2}(m_4+m_5(d+\frac{B}{2\Omega}+\frac{\sqrt{-\Omega}}{2\Psi}tan(\frac{\sqrt{-\Omega}\xi}{2A})$$
$$+m_6(d+\frac{B}{2\Omega}+\frac{\sqrt{-\Omega}}{2\Psi}tan(\frac{\sqrt{-\Omega}\xi}{2A})^{-2}),$$

$$\mathrm{u}_{65}(\xi)=\frac{2}{A^2}(m_4+m_5(d+\frac{B}{2\Omega}+\frac{C_2}{C_1+C_2\xi})^{-1}$$
$$+m_6(d+\frac{B}{2\Omega}+\frac{C_2}{C_1+C_2\xi})^{-2}),$$

$$u_{66}(\xi)=\frac{2}{A^2}(m_4+m_5(d+\frac{\sqrt{\Delta}}{\Psi}coth(\frac{\sqrt{\Delta}\xi}{2A})^{-1}+m_6(d$$
$$+\frac{\sqrt{\Delta}}{\Psi}coth(\frac{\sqrt{\Delta}\xi}{2A})^{-2}),$$

$$u_{67}(\xi)=\frac{2}{A^2}(m_4+m_5(d+\frac{\sqrt{\Delta}}{\Psi}tanh(\frac{\sqrt{\Delta}\xi}{2A})^{-1}+m_6(d$$
$$+\frac{\sqrt{\Delta}}{\Psi}tanh(\frac{\sqrt{\Delta}\xi}{2A})^{-2}),$$

$$u_{68}(\xi) = \frac{2}{A^2}(m_4 + m_5(d + \frac{\sqrt{-\Delta}}{\Psi}cot(\frac{\sqrt{-\Delta}\xi}{2A}))^{-1} + m_6(d + \frac{\sqrt{\Delta}}{\Psi}cot(\frac{\sqrt{-\Delta}\xi}{2A}))^{-2}),$$

$$u_{69}(\xi) = \frac{2}{A^2}(m_4 + m_5(d + \frac{\sqrt{-\Delta}}{\Psi}tan(\frac{\sqrt{-\Delta}\xi}{2A}))^{-1} + m_6(d + \frac{\sqrt{-\Delta}}{\Psi}tan(\frac{\sqrt{-\Delta}\xi}{2A}))^{-2}),$$

where $\xi = x + y - (\frac{\lambda A^2 + 4E\Psi + B^2}{A^2})t$.

Similarly for the seventh set, we obtain:

$$u_{71}(\xi) = \frac{1}{3A^2}(m_7 + 6m_8(d + \frac{B}{2\Omega} + \frac{\sqrt{\Omega}}{2\Psi}coth(\frac{\sqrt{\Omega}\xi}{2A})) + 6m_9(d + \frac{B}{2\Omega} + \frac{\sqrt{\Omega}}{2\Psi}coth(\frac{\sqrt{\Omega}\xi}{2A}))^{-2}),$$

$$u_{72}(\xi) = \frac{1}{3A^2}(m_7 + 6m_8(d + \frac{B}{2\Omega} + \frac{\sqrt{\Omega}}{2\Psi}tanh(\frac{\sqrt{\Omega}\xi}{2A})) + 6m_9(d + \frac{B}{2\Omega} + \frac{\sqrt{\Omega}}{2\Psi}tanh(\frac{\sqrt{\Omega}\xi}{2A}))^{-2}),$$

$$u_{73}(\xi) = \frac{1}{3A^2}(m_7 + 6m_8(d + \frac{B}{2\Omega} + \frac{\sqrt{-\Omega}}{2\Psi}cot(\frac{\sqrt{-\Omega}\xi}{2A})) + 6m_9(d + \frac{B}{2\Omega} + \frac{\sqrt{-\Omega}}{2\Psi}cot(\frac{\sqrt{-\Omega}\xi}{2A}))^{-2}),$$

$$u_{74}(\xi) = \frac{1}{3A^2}(m_7 + 6m_8(d + \frac{B}{2\Omega} + \frac{\sqrt{-\Omega}}{2\Psi}tan(\frac{\sqrt{-\Omega}\xi}{2A})) + 6m_9(d + \frac{B}{2\Omega} + \frac{\sqrt{-\Omega}}{2\Psi}tan(\frac{\sqrt{-\Omega}\xi}{2A}))^{-2}),$$

$$u_{75}(\xi) = \frac{1}{3A^2}(m_7 + 6m_8(d + \frac{B}{2\Omega} + \frac{C_2}{C_1 + C_2\xi})^{-1} + 6m_9(d + \frac{B}{2\Omega} + \frac{C_2}{C_1 + C_2\xi})^{-2}),$$

$$u_{76}(\xi) = \frac{1}{3A^2}(m_7 + m_8(d + \frac{\sqrt{\Delta}}{\Psi}coth(\frac{\sqrt{\Delta}\xi}{2A}))^{-1} + m_9(d + \frac{\sqrt{\Delta}}{\Psi}coth(\frac{\sqrt{\Delta}\xi}{2A}))^{-2}),$$

$$u_{77}(\xi) = \frac{1}{3A^2}(m_7 + m_8(d + \frac{\sqrt{\Delta}}{\Psi}tanh(\frac{\sqrt{\Delta}\xi}{2A}))^{-1} + m_9(d + \frac{\sqrt{\Delta}}{\Psi}tanh(\frac{\sqrt{\Delta}\xi}{2A}))^{-2}),$$

$$u_{78}(\xi) = \frac{1}{3A^2}(m_7 + m_8(d + \frac{\sqrt{-\Delta}}{\Psi}cot(\frac{\sqrt{-\Delta}\xi}{2A}))^{-1} + 6m_9(d + \frac{\sqrt{\Delta}}{\Psi}cot(\frac{\sqrt{\Delta}\xi}{2A}))^{-2}),$$

$$u_{79}(\xi) = \frac{1}{3A^2}(m_7 + 6m_8(d - \frac{\sqrt{-\Delta}}{\Psi}tan(\frac{\sqrt{-\Delta}\xi}{2A}))^{-1} + 6m_9(d - \frac{\sqrt{-\Delta}}{\Psi}tan(\frac{\sqrt{-\Delta}\xi}{2A}))^{-2}),$$

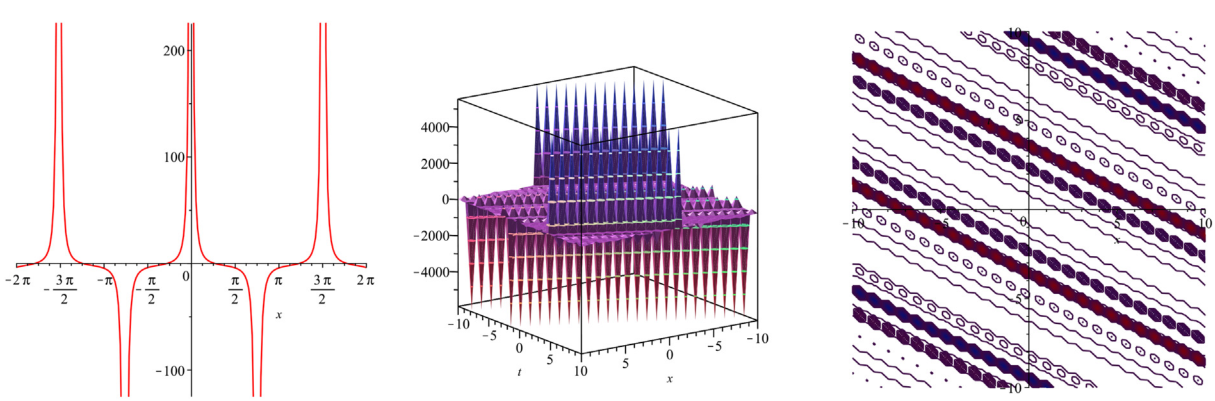
where $\xi = x + y - (\frac{\lambda A^2 - 4E\Psi - B^2}{A^2})t$.

4.1. Graphical representations

We have successfully obtained sixty-three exact wave solutions in terms of some free unknown constants of the studied equation through the novel generalized (G'/G)-expansion scheme. The obtained exact wave solutions are performed of rational, trigonometric and hyperbolic functions. If we change the appropriate values of the free unknown constants in each exact wave solutions, and next the solitary wave answers with compaction, cuspon, kink, periodic, soliton traveling wave solution, singular soliton traveling wave solution and various varieties of solutions may be succeeded. We have outlined some graphical illustration with $2D, 3D$ and the contour plot of the solitary wave answers by plugging the appropriate values of the free unknown constants. For more further assistance, the graphical depictions of $u_{14}(x,t), u_{22}(x,t), u_{55}(x,t), u_{61}(x,t)$ and $u_{64}(x,t)$ of studied equation are shown in Figs. 6–10, respectively.

5. Results and discussion

Wazwaz (Wazwaz, 2007c) solved the KP equation and received only eight wave solutions through the tanh-coth method. Contrary, in the current paper by plugging the novel generalized $(\frac{G'}{G})$-expansion process, numerous novel exact wave answers of the KP equation are constructed and also investigated three classes of explicit exact wave answers, for example, the hyperbolic, trigonometric and rational solutions under some unknown parameters. Comparing with Wazwaz (Wazwaz, 2007c) with our answers, we mentioned that (Wazwaz, 2007c) determined only trigonometric and hyperbolic types of solutions, but (Wazwaz, 2007c) did not solve any rational kind of solutions to the studied equation. On the contrary, we have determined rational, trigonometric and hyperbolic types of solutions to the considered equa-



a.$3D$ graph b. $2D$ graph c. Contour plot

Fig. 6. Graphical representation of $u_{14}(x,y,t)$ for $\lambda = 5, y = 0, A = 2, B = 1, C = 4,$ and $E = 1$ within the interval $-1 \leqslant x,t \leqslant 1$ and $t = 0.01$ for two dimensional.

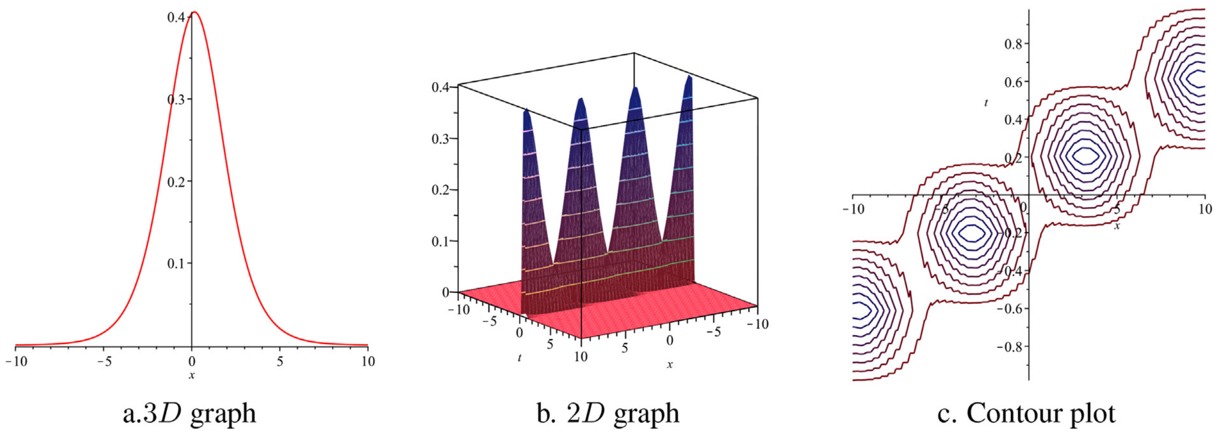


Fig. 7. Graphical representation of $u_{22}(x,y,t)$ for $\lambda = 15, y = 0, A = 4, B = 1, C = 1$, and $E = 1$ within the interval $-10 \leqslant x,t \leqslant 10$ and $t = 0.01$ for two dimensional.

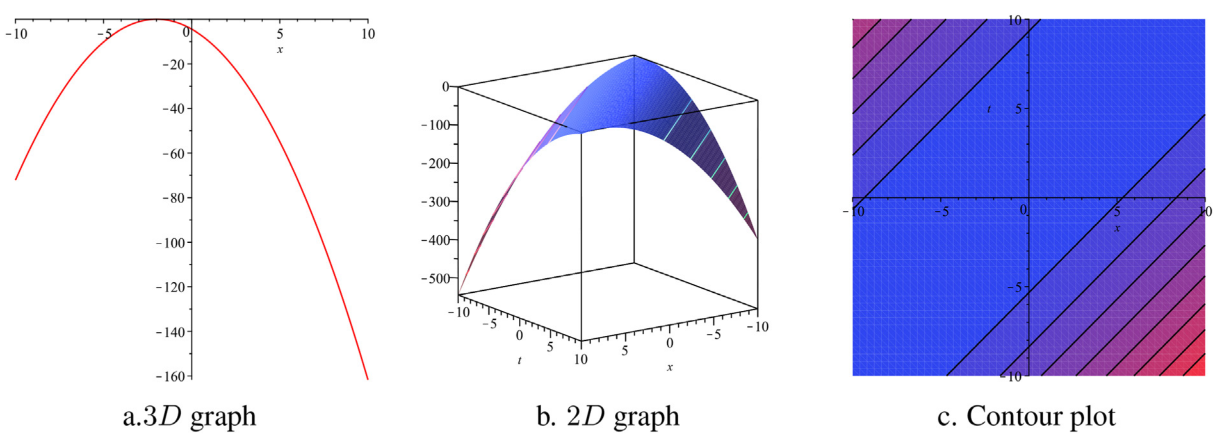


Fig. 8. Graphical representation of $u_{55}(x,y,t)$ for $\lambda = 1, C_1 = 1, C_2 = 2, y = 0, A = 1, B = 2, C = 2$, and $E = 2$ within the interval $-10 \leqslant x,t \leqslant 10$ and $t = 0.01$ for two dimensional.

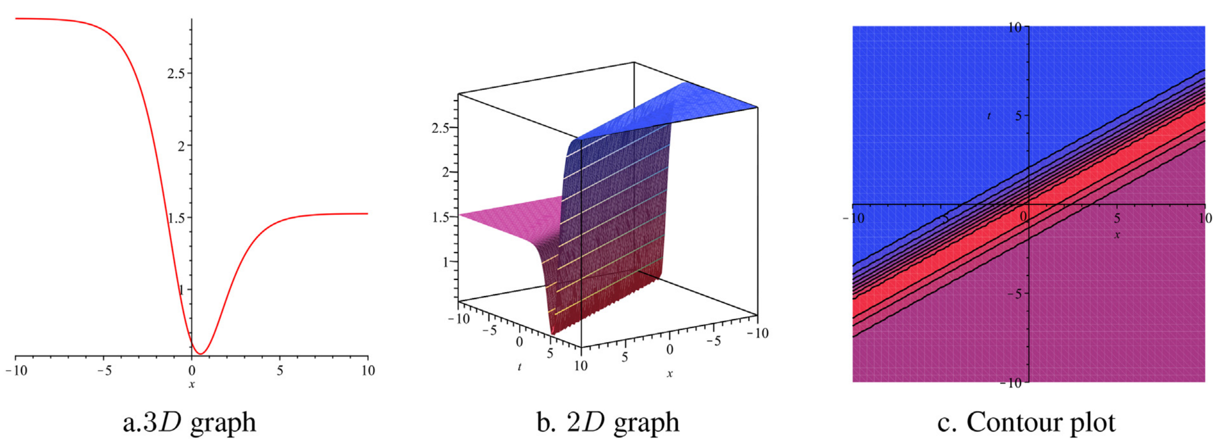


Fig. 9. Graphical representation of $u_{61}(x,y,t)$ for $\lambda = 1, d = 1, y = 0, A = 4, B = 1, C = 1$, and $E = 1$ within the interval $-10 \leqslant x,t \leqslant 10$ and $t = 0.01$ for two dimensional.

tion. From the observation with (Wazwaz, 2007c), all of the solutions are achieved in this paper are new and has not been seen in the earlier literature. We also mentioned that compaction, cuspon, bell shape soliton, kink, periodic, soliton, bright periodic wave, dark periodic wave and various varieties of soliton of the considered equation are achieved through the studied method which is shown in Figs. 6–10, respectively. These obtained solutions are the innovation and fulfilment of the present paper. We mention that few exact wave answers in the immediate investigation have a well-known physical application, like a liquid under small bubbles as well as turbulence (Fan et al., 2001), nonlinear wave models of fluid during an elastic tube as well as the acoustic dust waves

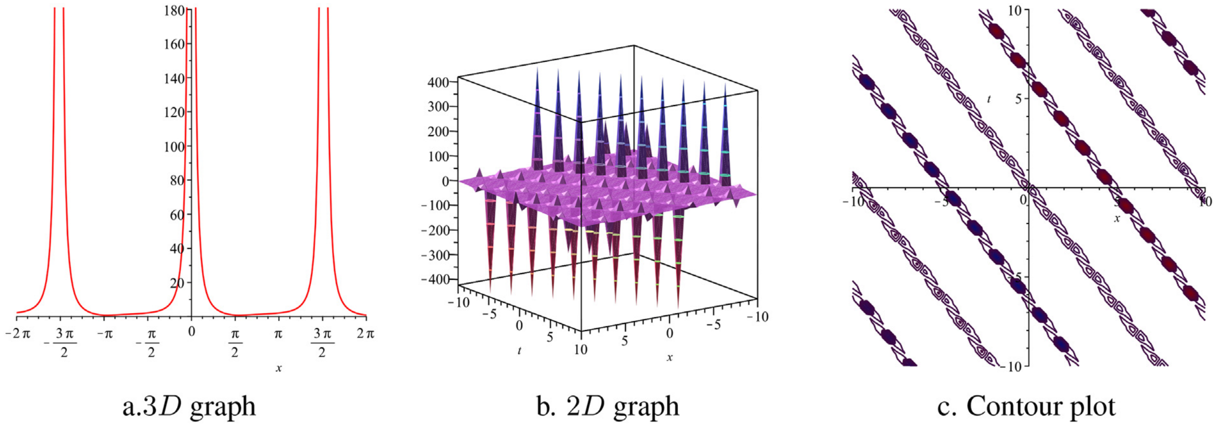


Fig. 10. Graphical representation of $u_{64}(x,y,t)$ for $\lambda=1, d=1, y=0, A=2, B=1, C=4$, and $E=1$ within the interval $-10\leqslant x,t\leqslant 10$ and $t=0.01$ for two dimensional.

within dusty plasmas under non-adiabatic dust charge fluctuation (Xue, 2003).

We actively investigated the novel generalized (G'/G) – expansion technique for ascertaining exact wave solutions of the KD and KP equations. The distinct variety of solitary wave answers such as compaction, bell shape soliton, cuspon, kink, periodic, soliton, bright periodic wave, dark periodic wave and various varieties of soliton are gained which implement in diverse fields of mathematical physics including fluid mechanics, nonlinear optics, quantum field theory, complex scalar nucleon field, and plasma physics. As our expected results, it may conclude that the investigated procedure is robust, sincere, and essential in giving numerous newly exact wave solutions of the different nonlinear wave shapes of PDEs. Subsequently, we would continue in our prospective studies.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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