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Discriminating between gamma and lognormal distributions with applications

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Abstract In this paper, we discuss the use of the coefficient of skewness as a goodness-of-fit test to distinguish between the gamma and lognormal distributions. We also show the limitations of this idea. Next, we use the moments of order statistics from gamma distribution to adjust the correlation goodness-of-fit test. In addition, we calculate the power of the test based on some other alterative distributions including the lognormal distribution. Further, we show some numerical illustration. Finally, we apply the procedure developed in the paper to some real data sets.

1. Introduction

Let X be a random variable has the three-parameter gamma [$Gamma(\theta, \lambda, \alpha)$] density function (pdf) as

$$f(x) = \frac{(x-\theta)^{\alpha-1}}{\lambda^{\alpha}\Gamma(\alpha)} \exp\left[-\left(\frac{x-\theta}{\lambda}\right)\right], \quad x \geqslant \theta, \lambda, \alpha > 0, \theta \geqslant 0, \tag{1.1}$$

where θ, λ and α are the location, scale and shape parameters, respectively. When $\theta = 0$, we have the pdf of the two-parameter gamma as:

$$f(x) = \frac{x^{\alpha-1}}{\lambda^{\alpha}\Gamma(\alpha)} \exp\left[-\frac{x}{\lambda}\right], \quad x \geqslant 0, \lambda, \alpha > 0, \tag{1.2}$$

when $\lambda = 1$ and $\theta = 0$, we have the pdf of the one-parameter gamma:

$$f(x) = \frac{x^{\alpha-1}}{\Gamma(\alpha)} \exp[-x], \quad x \geqslant 0, \alpha > 0. \tag{1.3}$$

Let Y be a two-parameter lognormal [$LN(\mu, \sigma)$] random variable with pdf

$$g(y) = \frac{1}{y\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{\log y - \mu}{2\sigma^2}\right)\right],$$
$$y \geqslant 0, \sigma > 0, -\infty < \mu < \infty. \tag{1.4}$$

For more details of the lognormal and gamma distributions, see Johnson et al. (1994). Some useful measures of the

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two-parameter gamma given in (1.2) and the two-parameter lognormal distributions given in (1.4) are listed below:

1. Mean:

$$E(X) = \alpha\lambda, \quad \text{and} \quad E(Y) = \exp[\mu + \sigma^2/2]. \tag{1.5}$$

2. Variance:

$$\begin{aligned} &\mathrm{Var}(X) = \alpha\lambda^2 \quad \text{and} \\ &\mathrm{Var}(Y) = \omega(1-\omega)\exp[2\mu], \quad \omega = \exp[\sigma^2]. \end{aligned} \tag{1.6}$$

3. Skewness:

$$SK(X) = \frac{2}{\alpha}, \quad \text{and} \quad SK(Y) = (\omega + 2)\sqrt{\omega - 1}. \tag{1.7}$$

The problem for testing whether some given data come from one of the two probability distributions, is quite old in the statistical literature. Atkinson (1969, 1970), Chen (1980), Chambers and Cox (1967), Cox (1961, 1962), Dyer (1973) have considered this problem in general for discriminating between two models. Due to increasing applications of the lifetime distributions, special attention is given to the problem of discriminating between the lognormal and Weibull distributions by Dumonceaux and Antle (1973) and between the lognormal and gamma by Jackson (1969) and between the gamma and Weibull distribution by Bain and Engelhard (1980) and Fearn and Nebenzahl (1991). Wiens (1999) has discussed a case study when the lognormal and gamma give different results. Recently, Gupta and Kundu (2003a) have discussed the closeness of gamma and the generalized exponential distribution while Gupta and Kundu (2003b) have discriminated between Weibull and the generalized exponential distributions. Gupta and Kundu (2004) have discriminated between gamma and the generalized exponential distribution.

On the other hand, goodness-of-fit tests are very important techniques for data analysis in the sense of check whether the given data fits the distributional assumptions of the statistical model. A variety of goodness-of-fit tests are available in the literature and recently there seems to be significant research on this topic. For more details, see, D'Agostino and Stephens (1986) and Huber-Carol et al. (2002). Correlation coefficient test is considered one of the easiest of such tests, that is because it is only needs special tables introduce from Monte Carlo simulations. The correlation coefficient test was introduced by Filliben (1975) for testing goodness-of-fit to the normal distribution and tables where updated later by Looney and Gulledge (1985). Among others Kinnison (1985, 1989) used the correlation coefficient method to present tables for testing goodness-of-fit to the extreme-value Type-I (Gumbel) and the extreme-value distribution, respectively. Recently, Sultan (2001) has devolved the correlation goodness-of-fit to the logarithmically-decreasing survival distribution. Baklizi (2006) has suggested weighted Kolmogrove–Smirnov type test for grouped Rayleigh data. Chen (2006) has discussed some tests of fit for the three-parameter lognormal distribution.

In this paper, we discuss the motivation of the problem in Section 2 below. In Section 3, we use the single moments of the rth order statistic from the one-parameter gamma distribution to develop goodness-of-fit tests for the two- and three-parameter gamma distributions. In Section 4, we calculate the power of the tests based on some different alternative distributions. In addition, we discuss some simulated examples. Finally, in Section 5, we apply the proposed test for some real data sets were collected from Dalla hospital, Riyadh, Saudi Arabia.

2. Motivation

The problem starts whenever we have a certain data and we need to fit the given data to either gamma or lognormal distributions. In many situations, we have found that gamma distribution fits better than the lognormal distribution. Then a question rises: why we do use the lognormal? Consequently, the answer of such question leads us to discuss some issues they are: (i) different measures of skewness, (ii) nonparametric tests, and (iii) correlation coefficient goodness-of-fit test.

Let $X_1, \ldots, X_n$ be a random sample has mean M and variance V and assume:

$$E(X) = E(Y) = M,$$

and

$$Var(X) = Var(Y) = V,$$

then it is easy to write

$$\alpha = \frac{M^2}{V} \quad \text{and} \quad \lambda = \frac{V}{M}. \tag{2.1}$$

Similarly, we write:

$$\mu = \log\left(\frac{M^2}{\sqrt{M^2 + V}}\right) \quad \text{and} \quad \sigma = \sqrt{\log\left(\frac{V + M^2}{V}\right)}. \tag{2.2}$$

2.1. Result 1

If $E(X) = E(Y)$ and $\mathrm{Var}(X) = \mathrm{Var}(Y)$, then by using (1.7), (2.1) and (2.2), we have

$$SK(X) < SK(Y).$$

It thought that Results 1 could be used to distinguish between gamma and lognormal distributions by calculating the skewness for the given data. Then the closer values of the skewness to either of $SK(X)$ and $SK(Y)$ fits the given data. Unfortunately, this approach has some limitations based on the mean and variance for the given data. Among 10,000 Monte Carlo simulations, this approach works out well when the mean of the given data is less than 2.8 and the variance is greater than 3.

This is also true when we apply the nonparametric tests such as chi-square and Kolmogorov–Smirnov tests. So, we use the correlation goodness-of-fit tests.

3. Correlation goodness of fit test of gamma pdf

Let $x_{1:n}, \ldots, x_{n-r:n}$ represents n order statistics from $Gamma(0, 1, \alpha)$ given in (1.3). Then, the pdf of the rth order statistic is given by:

$$f_{r:n}(x) = \frac{n!}{(r-1)!(n-r)!}[F(x)]^{r-1}[1 - F(x)]^{n-r}f(x),$$
$$r = 1, 2, \ldots, n. \tag{3.1}$$

For more details, see David (1981), David and Nagaraja (2003) and Arnold et al. (1992). The single moment of the rth order statistic is given by:

$$\mu_{r:n}^{(k)} = \int_0^\infty x^k f_{r:n}(x)dx. \tag{3.2}$$

Gupta (1960, 1962) has derived the first single moments of the rth order statistic from gamma distribution in (1.3) when the shape parameter α is integer as follows:

$$\mu_{r:n}^{(k)} = \frac{n!}{(r-1)!(n-r)!\Gamma(\alpha)} \sum_{j=0}^{r-1} (-1)^j \binom{r-1}{j} \times \sum_{s=0}^{(\alpha-1)(n-r+j)} a_s(\alpha, n-r+j)\Gamma(k+\alpha+s) \times \frac{1}{(n-r+j+1)^{k+\alpha+s}}, \tag{3.3}$$

where $a_s(\alpha, n-r+j)$ is the coefficient of x^s in the expansion of $\left(\sum_{\ell=0}^{\alpha-1} \frac{x^\ell}{\ell!}\right)^{n-r+j}$.

3.1. Test for the two-parameter case

Let $X_{1:n}, \ldots, X_{n-r:n}$ denote a Type-II right-censored sample from the gamma distribution in (1.2), and let $Z_{i:n} = X_{i:n}/\lambda$, $i = 1, 2, n-r$, be the corresponding order statistics from the one-parameter gamma in (1.3). Let us denote:

$E(Z_{i:n})$ by $\mu_{i:n}$, the $E(X_{i:n}) = \lambda\mu_{i:n}$, $\quad i = 1, 2, \ldots, n-r$.

The correlation goodness-of-fit test in this case may be formed as follows:

H_0: F is correct, that is $X_1, X_2, \ldots, X_n$ have $Gamma(0, \lambda, \alpha)$ given in (1.2) versus, H_1: Fist not correct, that is $X_1, X_2, \ldots, X_n$ have another pdf, and the statistic used to run the test is given by:

$$T_1 = \sum_{i=1}^{n-r} X_{i:n}\mu_{i:n} \Bigg/ \sqrt{\sum_{i=1}^{n-r} X_{i:n}^2 \sum_{i=1}^{n-r} \mu_{i:n}^2}. \tag{3.4}$$

This statistic represents the correlation between $X_{i:n}$ and $\mu_{i:n}$, $i = 1, 2, \ldots, n-r$. By using the formula of the moments $\mu_{i:n}$ obtained in (3.3), and using the IMSL package, the statistic T_1 is simulated through Monte Carlo method based on 10,001 simulations. Table 1 represents the percentage points of T_1 for sample sizes up to $n = 25$ and different censoring ratios $p = \frac{n-r}{n} = 1.0, 0.8, 0.6$.

As we can see from Table 1, the percentage points of T_1 increases as the sample size increases as well as the significance level increases for censoring rations $p = 1.0, 0.8, 0.6$.

3.2. Test for the three-parameter case

Let $X_{1:n}, \ldots, X_{n-r:n}$ denote a Type-II right-censored sample from the distribution in (1.1), and let $Z_i = X_{i+1} - X_{1:n}$ and $U_i = \mu_{i+1:n} - \mu_{1:n}$, $i = 1, 2, \ldots, n-r-1$, where $\mu_{i:n}$ be the corresponding moments of order statistics obtained from $Gamma(0, 1, \alpha)$ given in (1.3). The correlation goodness-of-fit test in this case may be formed as follows:

H_0: F is correct, that is $X_1, X_2, \ldots, X_n$ have $Gamma(\theta, \lambda, \alpha)$ given in (1.1) versus,
H_1: F is not correct, that is $X_1, X_2, \ldots, X_n$ have another pdf.

The statistic used to run the test is given by:

$$T_2 = \sum_{i=1}^{n-r-1} Z_i U_i \Bigg/ \sqrt{\sum_{i=1}^{n-r-1} Z_i^2 \sum_{i=1}^{n-r-1} U_i^2}. \tag{3.5}$$

The statistic given in (3.5) represents the correlation between $Z_{i:n}$ and $U_{i:n}$, $i = 1, 2, \ldots, n-r$. Once again, by using the formula of the moments $\mu_{i:n}$, $i = 1, 2, \ldots, n-r$ given in (3.3), the statistic T_2 is simulated through Monte Carlo method based on 10,001 simulations. Table 2 represents the percentage points of T_2 for sample sizes $n = 10, 20, 30, 40, 50$ and different censoring ratios p.

4. Power calculation

In this section, we calculate the power of the considered tests by replacing the $Gamma(\theta, \lambda, \alpha)$ random variates generator in the simulation program with generators from the alternatives including; normal, lognormal, and the Weibull distributions. Based on different sample size, different censoring ratios and 10,001 simulations, the power is calculated to be

$$Power = \frac{\#\ of\ rejection\ of\ H_0}{10,001},$$

where H_0 is rejected if T_1 (T_2) $\geqslant$ the corresponding percentage points given in Table 1 (Table 2), and T_1 (T_2) is evaluated from the alternative distributions.

Tables 3 and 4 represent the power of the test for the two-parameters and three-parameter cases, respectively. The different considered alternative distributions are:

1. Normal distribution $N(\mu, \sigma)$.
2. Lognormal distribution $LN(\mu, \sigma)$.
3. Weibull distribution with shape α, scale parameter σ and location parameter μ, $W(\mu, \sigma, \alpha)$.
4. Chi-square distribution $\chi^2(\alpha)$.
5. Cauchy distribution with scale parameter σ and location parameter σ, $C(\mu, \sigma)$.
6. Mixtures of two exponential distribution $MTE(\theta_1, \theta_2, w) = wf_1(\theta_1) + (1-w)f_2(\theta_2)$.

Tables 3 and 4 indicate that the correlation test has good power to reject sample from the chosen alternative distributions. Also, the power increases as the sample sizes increase for all given censoring ratios $p = 1.0, 0.8, 0.6$ as well as the significance level increases.

4.1. Examples

In order to illustrate and show the performance of the correlation coefficient goodness-of-fit test for gamma distribution in both cases (two-parameter and three-parameter), we simulate four sets of order statistics each of size 20; they are

1. Sample from $LN(0, 1)$: one-parameter case of the lognormal distribution with $\mu = 0$ and $\sigma = 1$.
2. Sample from $Gamma(0, 1, 2)$: two-parameter gamma distribution with location parameter is equal to 0, scale parameter is equal to 1 and shape parameter is equal to 2.
3. Sample from $Gamma(1, 5, 3)$: three-parameter cases of gamma distribution with location parameter is equal to 1, scale parameter is equal to 3 and shape parameter is equal to 3.

Table 1 The lower percentage points of T_1.

α	p	n	0.5%	1%	2%	2.5%	5%	10%	20%	30%	40%	50%
2	1.0	10	0.912	0.928	0.941	0.946	0.958	0.968	0.977	0.982	0.985	0.988
		20	0.937	0.951	0.960	0.963	0.971	0.979	0.985	0.988	0.990	0.991
		30	0.952	0.961	0.968	0.971	0.978	0.983	0.988	0.990	0.992	0.993
		40	0.958	0.967	0.974	0.976	0.982	0.986	0.990	0.992	0.993	0.994
		50	0.965	0.971	0.978	0.980	0.984	0.988	0.992	0.993	0.994	0.995
3		10	0.925	0.941	0.956	0.959	0.968	0.976	0.982	0.986	0.988	0.990
		20	0.951	0.962	0.970	0.973	0.979	0.984	0.989	0.991	0.992	0.994
		30	0.967	0.972	0.978	0.979	0.984	0.988	0.991	0.993	0.994	0.995
		40	0.972	0.977	0.982	0.983	0.987	0.990	0.993	0.994	0.995	0.996
		50	0.974	0.980	0.984	0.985	0.989	0.992	0.994	0.995	0.996	0.997
4		10	0.946	0.955	0.964	0.967	0.974	0.980	0.986	0.989	0.991	0.992
		20	0.962	0.969	0.977	0.979	0.983	0.987	0.991	0.993	0.994	0.995
		30	0.973	0.978	0.982	0.984	0.988	0.991	0.993	0.994	0.995	0.996
		40	0.976	0.981	0.986	0.987	0.990	0.992	0.995	0.996	0.996	0.997
		50	0.981	0.985	0.988	0.989	0.992	0.994	0.995	0.996	0.997	0.997
5		10	0.953	0.962	0.970	0.973	0.979	0.984	0.988	0.991	0.992	0.993
		20	0.969	0.975	0.980	0.982	0.986	0.989	0.992	0.994	0.995	0.996
		30	0.979	0.982	0.986	0.987	0.990	0.992	0.994	0.995	0.996	0.997
		40	0.982	0.985	0.989	0.990	0.992	0.994	0.995	0.996	0.997	0.997
		50	0.985	0.988	0.990	0.991	0.993	0.995	0.996	0.997	0.997	0.998
2	0.8	10	0.933	0.945	0.954	0.957	0.967	0.974	0.982	0.986	0.988	0.990
		20	0.966	0.970	0.976	0.977	0.982	0.986	0.990	0.992	0.993	0.994
		30	0.978	0.982	0.984	0.985	0.988	0.991	0.993	0.994	0.995	0.996
		40	0.983	0.986	0.988	0.989	0.991	0.993	0.995	0.996	0.996	0.997
		50	0.987	0.989	0.990	0.991	0.993	0.994	0.996	0.996	0.997	0.998
3		10	0.952	0.960	0.967	0.969	0.975	0.981	0.986	0.989	0.991	0.993
		20	0.976	0.979	0.982	0.984	0.987	0.990	0.992	0.994	0.995	0.996
		30	0.984	0.986	0.988	0.989	0.991	0.993	0.995	0.996	0.996	0.997
		40	0.988	0.990	0.991	0.992	0.993	0.995	0.996	0.997	0.997	0.998
		50	0.990	0.992	0.993	0.993	0.995	0.996	0.997	0.997	0.998	0.998
4		10	0.962	0.968	0.973	0.975	0.980	0.985	0.989	0.991	0.993	0.994
		20	0.981	0.984	0.986	0.987	0.990	0.992	0.994	0.995	0.996	0.997
		30	0.987	0.989	0.991	0.992	0.993	0.995	0.996	0.997	0.997	0.998
		40	0.990	0.992	0.993	0.993	0.995	0.996	0.997	0.997	0.998	0.998
		50	0.992	0.993	0.994	0.995	0.996	0.997	0.997	0.998	0.998	0.998
5		10	0.967	0.972	0.977	0.979	0.983	0.987	0.991	0.993	0.994	0.995
		20	0.984	0.986	0.988	0.989	0.991	0.993	0.995	0.996	0.997	0.997
		30	0.989	0.991	0.992	0.993	0.994	0.995	0.996	0.997	0.998	0.998
		40	0.992	0.993	0.994	0.995	0.996	0.996	0.997	0.998	0.998	0.998
		50	0.994	0.995	0.995	0.996	0.996	0.997	0.998	0.998	0.999	0.999
2	0.6	10	0.932	0.942	0.952	0.955	0.965	0.973	0.980	0.985	0.987	0.990
		20	0.962	0.968	0.974	0.975	0.981	0.985	0.989	0.991	0.993	0.994
		30	0.975	0.978	0.982	0.983	0.986	0.989	0.992	0.994	0.995	0.996
		40	0.980	0.984	0.987	0.988	0.990	0.992	0.994	0.995	0.996	0.997
		50	0.985	0.988	0.989	0.990	0.992	0.994	0.995	0.996	0.997	0.997
3		10	0.942	0.953	0.964	0.966	0.973	0.980	0.985	0.988	0.991	0.992
		20	0.972	0.976	0.981	0.982	0.986	0.989	0.992	0.993	0.995	0.995
		30	0.981	0.984	0.987	0.988	0.990	0.992	0.994	0.995	0.996	0.997
		40	0.986	0.988	0.990	0.991	0.992	0.994	0.996	0.996	0.997	0.998
		50	0.989	0.991	0.992	0.993	0.994	0.995	0.996	0.997	0.998	0.998
4		10	0.956	0.964	0.971	0.973	0.979	0.984	0.989	0.991	0.993	0.994
		20	0.979	0.982	0.985	0.986	0.989	0.991	0.993	0.995	0.996	0.996
		30	0.986	0.988	0.990	0.990	0.992	0.994	0.995	0.996	0.997	0.997
		40	0.989	0.991	0.992	0.993	0.994	0.995	0.997	0.997	0.998	0.998
		50	0.991	0.993	0.994	0.994	0.995	0.996	0.997	0.998	0.998	0.998
5		10	0.965	0.970	0.976	0.978	0.983	0.987	0.991	0.993	0.994	0.995
		20	0.982	0.985	0.988	0.988	0.990	0.993	0.995	0.996	0.996	0.997
		30	0.988	0.990	0.992	0.992	0.994	0.995	0.996	0.997	0.997	0.998
		40	0.991	0.992	0.994	0.994	0.995	0.996	0.997	0.998	0.998	0.998
		50	0.993	0.994	0.995	0.995	0.996	0.997	0.998	0.998	0.998	0.999

Table 2 The lower percentage points of T_2.

α	p	n	0.5%	1%	2%	2.5%	5%	10%	20%	30%	40%	50%
2	1.0	10	0.901	0.914	0.929	0.934	0.947	0.960	0.971	0.977	0.981	0.985
		20	0.928	0.940	0.953	0.955	0.967	0.975	0.982	0.985	0.988	0.990
		30	0.944	0.956	0.964	0.967	0.975	0.981	0.986	0.989	0.991	0.992
		40	0.956	0.963	0.971	0.974	0.979	0.984	0.989	0.991	0.992	0.994
		50	0.962	0.969	0.975	0.977	0.982	0.987	0.990	0.992	0.994	0.995
3		10	0.905	0.917	0.934	0.938	0.952	0.963	0.973	0.979	0.983	0.986
		20	0.935	0.947	0.958	0.961	0.970	0.978	0.984	0.987	0.989	0.991
		30	0.950	0.961	0.969	0.972	0.978	0.983	0.988	0.990	0.992	0.993
		40	0.961	0.969	0.975	0.977	0.982	0.987	0.990	0.992	0.994	0.995
		50	0.967	0.973	0.978	0.980	0.985	0.989	0.992	0.994	0.995	0.995
4		10	0.904	0.921	0.936	0.941	0.955	0.965	0.975	0.980	0.983	0.986
		20	0.942	0.952	0.962	0.964	0.972	0.979	0.985	0.988	0.990	0.992
		30	0.954	0.964	0.972	0.973	0.980	0.985	0.989	0.991	0.993	0.994
		40	0.964	0.971	0.977	0.979	0.984	0.988	0.991	0.993	0.994	0.995
		50	0.971	0.977	0.981	0.983	0.987	0.990	0.993	0.994	0.995	0.996
5		10	0.910	0.926	0.940	0.944	0.956	0.967	0.975	0.980	0.984	0.987
		20	0.945	0.954	0.963	0.966	0.974	0.980	0.985	0.988	0.990	0.992
		30	0.957	0.966	0.972	0.975	0.981	0.986	0.990	0.992	0.993	0.994
		40	0.966	0.974	0.979	0.980	0.985	0.988	0.992	0.993	0.995	0.995
		50	0.972	0.978	0.982	0.983	0.987	0.990	0.993	0.994	0.995	0.996
2	0.8	10	0.906	0.922	0.936	0.941	0.953	0.964	0.974	0.979	0.983	0.986
		20	0.953	0.961	0.967	0.969	0.975	0.981	0.986	0.989	0.991	0.992
		30	0.971	0.975	0.979	0.980	0.984	0.988	0.991	0.993	0.994	0.995
		40	0.977	0.981	0.984	0.986	0.988	0.991	0.993	0.995	0.996	0.996
		50	0.983	0.986	0.988	0.989	0.991	0.993	0.995	0.996	0.996	0.997
3		10	0.906	0.921	0.937	0.941	0.954	0.966	0.976	0.981	0.985	0.987
		20	0.956	0.964	0.969	0.972	0.977	0.982	0.987	0.990	0.992	0.993
		30	0.971	0.975	0.979	0.980	0.985	0.988	0.992	0.993	0.994	0.995
		40	0.978	0.981	0.985	0.986	0.989	0.991	0.994	0.995	0.996	0.997
		50	0.984	0.986	0.988	0.989	0.991	0.993	0.995	0.996	0.997	0.997
4		10	0.897	0.919	0.936	0.941	0.954	0.966	0.976	0.981	0.985	0.987
		20	0.955	0.963	0.969	0.971	0.977	0.983	0.988	0.990	0.992	0.993
		30	0.971	0.977	0.981	0.982	0.985	0.989	0.992	0.994	0.995	0.996
		40	0.979	0.982	0.985	0.986	0.989	0.992	0.994	0.995	0.996	0.997
		50	0.984	0.986	0.988	0.989	0.991	0.993	0.995	0.996	0.997	0.997
5		10	0.907	0.922	0.935	0.941	0.954	0.966	0.976	0.982	0.985	0.988
		20	0.958	0.963	0.969	0.972	0.978	0.983	0.988	0.991	0.992	0.994
		30	0.973	0.977	0.981	0.982	0.986	0.989	0.992	0.994	0.995	0.996
		40	0.977	0.982	0.985	0.986	0.989	0.992	0.994	0.995	0.996	0.997
		50	0.982	0.985	0.988	0.989	0.991	0.993	0.995	0.996	0.997	0.997
2	0.6	10	0.879	0.901	0.919	0.925	0.943	0.958	0.970	0.977	0.982	0.985
		20	0.940	0.949	0.959	0.962	0.971	0.978	0.985	0.988	0.990	0.992
		30	0.965	0.970	0.975	0.976	0.981	0.985	0.989	0.992	0.993	0.994
		40	0.974	0.979	0.982	0.983	0.986	0.990	0.992	0.994	0.995	0.996
		50	0.979	0.982	0.985	0.987	0.989	0.992	0.994	0.995	0.996	0.997
3		10	0.884	0.900	0.920	0.928	0.945	0.959	0.971	0.978	0.982	0.985
		20	0.942	0.953	0.962	0.965	0.973	0.979	0.985	0.988	0.990	0.992
		30	0.963	0.970	0.975	0.977	0.982	0.986	0.990	0.992	0.994	0.995
		40	0.974	0.977	0.981	0.983	0.986	0.990	0.992	0.994	0.995	0.996
		50	0.979	0.982	0.985	0.986	0.990	0.992	0.994	0.995	0.996	0.997
4		10	0.885	0.903	0.923	0.928	0.945	0.960	0.972	0.978	0.983	0.986
		20	0.941	0.952	0.960	0.963	0.972	0.979	0.985	0.988	0.991	0.992
		30	0.963	0.970	0.975	0.977	0.982	0.986	0.990	0.992	0.994	0.995
		40	0.974	0.978	0.982	0.983	0.986	0.990	0.993	0.994	0.995	0.996
		50	0.980	0.983	0.986	0.987	0.990	0.992	0.994	0.995	0.996	0.997
5		10	0.884	0.903	0.921	0.927	0.945	0.960	0.972	0.979	0.983	0.986
		20	0.943	0.952	0.961	0.964	0.972	0.979	0.986	0.989	0.991	0.993
		30	0.963	0.970	0.975	0.976	0.981	0.986	0.990	0.992	0.994	0.995
		40	0.972	0.976	0.981	0.982	0.986	0.990	0.993	0.994	0.995	0.996
		50	0.979	0.982	0.985	0.987	0.989	0.992	0.994	0.995	0.996	0.997

Table 3 Power of the test based on the two-parameter case.

α	p	n	N(0,1)		W(0,1,5)		W(0,1,10)		LN(0,1)	
			5%	10%	5%	10%	5%	10%	5%	10%
2	1.0	10	0.994	0.997	0.939	0.987	1.000	1.000	0.371	0.456
		20	1.000	1.000	1.000	1.000	1.000	1.000	0.566	0.657
		30	1.000	1.000	1.000	1.000	1.000	1.000	0.693	0.769
		40	1.000	1.000	1.000	1.000	1.000	1.000	0.793	0.853
		50	1.000	1.000	1.000	1.000	1.000	1.000	0.859	0.908
3		10	0.999	0.999	0.748	0.915	1.000	1.000	0.603	0.685
		20	1.000	1.000	0.997	1.000	1.000	1.000	0.828	0.885
		30	1.000	1.000	1.000	1.000	1.000	1.000	0.937	0.963
		40	1.000	1.000	1.000	1.000	1.000	1.000	0.976	0.989
		50	1.000	1.000	1.000	1.000	1.000	1.000	0.992	0.997
4		10	0.999	0.999	0.534	0.766	0.998	1.000	0.772	0.832
		20	1.000	1.000	0.958	0.993	1.000	1.000	0.945	0.968
		30	1.000	1.000	0.999	1.000	1.000	1.000	0.989	0.996
		40	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.999
		50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5		10	1.000	1.000	0.361	0.601	0.993	1.000	0.867	0.908
		20	1.000	1.000	0.852	0.957	1.000	1.000	0.982	0.991
		30	1.000	1.000	0.988	0.999	1.000	1.000	0.998	0.999
		40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.8	10	0.999	0.999	0.695	0.867	0.998	1.000	0.206	0.287
		20	1.000	1.000	0.995	0.999	1.000	1.000	0.301	0.394
		30	1.000	1.000	1.000	1.000	1.000	1.000	0.402	0.501
		40	1.000	1.000	1.000	1.000	1.000	1.000	0.472	0.575
		50	1.000	1.000	1.000	1.000	1.000	1.000	0.554	0.642
3		10	0.999	0.999	0.426	0.651	0.979	0.997	0.452	0.545
		20	1.000	1.000	0.936	0.976	1.000	1.000	0.680	0.756
		30	1.000	1.000	0.997	0.999	1.000	1.000	0.812	0.867
		40	1.000	1.000	1.000	1.000	1.000	1.000	0.894	0.929
		50	1.000	1.000	1.000	1.000	1.000	1.000	0.941	0.967
4		10	1.000	1.000	0.261	0.464	0.917	0.984	0.648	0.725
		20	1.000	1.000	0.774	0.892	1.000	1.000	0.876	0.915
		30	1.000	1.000	0.960	0.985	1.000	1.000	0.962	0.976
		40	1.000	1.000	0.994	0.998	1.000	1.000	0.984	0.991
		50	1.000	1.000	0.999	1.000	1.000	1.000	0.995	0.998
5		10	1.000	1.000	0.151	0.336	0.817	0.953	0.760	0.827
		20	1.000	1.000	0.565	0.725	1.000	1.000	0.951	0.969
		30	1.000	1.000	0.830	0.914	1.000	1.000	0.990	0.994
		40	1.000	1.000	0.949	0.980	1.000	1.000	0.998	0.999
		50	1.000	1.000	0.984	0.995	1.000	1.000	1.000	1.000
2	0.6	10	1.000	1.000	0.351	0.593	0.903	0.980	0.114	0.179
		20	1.000	1.000	0.914	0.970	1.000	1.000	0.155	0.231
		30	1.000	1.000	0.994	0.999	1.000	1.000	0.176	0.260
		40	1.000	1.000	1.000	1.000	1.000	1.000	0.204	0.290
		50	1.000	1.000	1.000	1.000	1.000	1.000	0.226	0.314
3		10	0.999	0.999	0.147	0.355	0.674	0.896	0.291	0.389
		20	1.000	1.000	0.642	0.807	0.999	1.000	0.443	0.542
		30	1.000	1.000	0.906	0.962	1.000	1.000	0.564	0.657
		40	1.000	1.000	0.978	0.994	1.000	1.000	0.652	0.742
		50	1.000	1.000	0.997	0.999	1.000	1.000	0.747	0.818
4		10	1.000	1.000	0.072	0.210	0.482	0.772	0.483	0.580
		20	1.000	1.000	0.391	0.574	0.990	0.998	0.687	0.768
		30	1.000	1.000	0.676	0.809	1.000	1.000	0.827	0.881
		40	1.000	1.000	0.858	0.936	1.000	1.000	0.903	0.939
		50	1.000	1.000	0.948	0.979	1.000	1.000	0.952	0.971
5		10	1.000	1.000	0.043	0.133	0.353	0.638	0.622	0.696
		20	1.000	1.000	0.236	0.395	0.952	0.989	0.837	0.884
		30	1.000	1.000	0.483	0.636	1.000	1.000	0.937	0.958
		40	1.000	1.000	0.638	0.781	1.000	1.000	0.974	0.985
		50	1.000	1.000	0.783	0.882	1.000	1.000	0.991	0.995

Table 4 Power of the test based on the three-parameter case.

α	p	n	LN(0,1)		χ²(1)		MTE(4,2,0.5)		C(0,1)	
			5%	10%	5%	10%	5%	10%	5%	10%
2	1	10	0.985	0.992	0.368	0.483	0.628	0.735	0.589	0.678
		20	1.000	1.000	0.684	0.788	0.916	0.961	0.873	0.918
		30	1.000	1.000	0.864	0.923	0.988	0.997	0.964	0.979
		40	1.000	1.000	0.944	0.975	0.999	1.000	0.990	0.996
		50	1.000	1.000	0.975	0.992	1.000	1.000	0.997	0.999
3		10	0.993	0.996	0.496	0.613	0.745	0.831	0.579	0.670
		20	1.000	1.000	0.844	0.911	0.974	0.990	0.863	0.913
		30	1.000	1.000	0.964	0.983	0.999	1.000	0.955	0.976
		40	1.000	1.000	0.993	0.998	1.000	1.000	0.988	0.994
		50	1.000	1.000	0.999	1.000	1.000	1.000	0.996	0.998
4		10	0.996	0.998	0.578	0.684	0.803	0.878	0.574	0.665
		20	1.000	1.000	0.903	0.946	0.989	0.995	0.856	0.902
		30	1.000	1.000	0.986	0.994	1.000	1.000	0.953	0.973
		40	1.000	1.000	0.999	1.000	1.000	1.000	0.986	0.994
		50	1.000	1.000	1.000	1.000	1.000	1.000	0.996	0.998
5		10	0.996	0.999	0.623	0.726	0.835	0.902	0.565	0.660
		20	1.000	1.000	0.938	0.968	0.994	0.998	0.853	0.899
		30	1.000	1.000	0.993	0.998	1.000	1.000	0.951	0.971
		40	1.000	1.000	1.000	1.000	1.000	1.000	0.986	0.993
		50	1.000	1.000	1.000	1.000	1.000	1.000	0.995	0.998
2	0.8	10	0.947	0.967	0.389	0.492	0.662	0.745	0.491	0.592
		20	0.999	1.000	0.740	0.818	0.961	0.974	0.868	0.907
		30	1.000	1.000	0.916	0.952	0.996	0.998	0.969	0.980
		40	1.000	1.000	0.976	0.989	0.999	1.000	0.993	0.995
		50	1.000	1.000	0.995	0.999	1.000	1.000	0.999	1.000
3		10	0.964	0.978	0.471	0.585	0.729	0.806	0.432	0.546
		20	1.000	1.000	0.852	0.903	0.980	0.988	0.825	0.872
		30	1.000	1.000	0.971	0.987	0.998	0.999	0.948	0.965
		40	1.000	1.000	0.996	0.999	1.000	1.000	0.986	0.991
		50	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.998
4		10	0.969	0.982	0.512	0.624	0.757	0.827	0.390	0.510
		20	1.000	1.000	0.888	0.935	0.986	0.993	0.786	0.847
		30	1.000	1.000	0.984	0.993	0.999	0.999	0.929	0.951
		40	1.000	1.000	0.999	0.999	1.000	1.000	0.979	0.986
		50	1.000	1.000	1.000	1.000	1.000	1.000	0.994	0.996
5		10	0.972	0.985	0.536	0.649	0.774	0.844	0.363	0.485
		20	1.000	1.000	0.913	0.951	0.989	0.995	0.765	0.828
		30	1.000	1.000	0.991	0.996	0.999	0.999	0.918	0.939
		40	1.000	1.000	0.999	1.000	1.000	1.000	0.970	0.981
		50	1.000	1.000	1.000	1.000	1.000	1.000	0.990	0.994
2	0.6	10	0.786	0.858	0.299	0.407	0.388	0.481	0.299	0.438
		20	0.993	0.996	0.655	0.751	0.704	0.777	0.780	0.851
		30	1.000	1.000	0.859	0.909	0.855	0.895	0.944	0.964
		40	1.000	1.000	0.946	0.968	0.932	0.957	0.987	0.993
		50	1.000	1.000	0.983	0.992	0.969	0.981	0.998	0.999
3		10	0.833	0.889	0.361	0.477	0.442	0.541	0.250	0.388
		20	0.996	0.998	0.755	0.828	0.781	0.839	0.722	0.793
		30	1.000	1.000	0.924	0.957	0.912	0.942	0.904	0.939
		40	1.000	1.000	0.978	0.989	0.969	0.982	0.968	0.980
		50	1.000	1.000	0.997	0.998	0.990	0.995	0.991	0.996
4		10	0.850	0.903	0.389	0.509	0.466	0.566	0.211	0.354
		20	0.997	0.999	0.782	0.859	0.803	0.863	0.659	0.756
		30	1.000	1.000	0.946	0.970	0.933	0.957	0.873	0.913
		40	1.000	1.000	0.988	0.994	0.980	0.989	0.952	0.970
		50	1.000	1.000	0.998	0.999	0.995	0.997	0.986	0.991
5		10	0.861	0.908	0.404	0.523	0.479	0.580	0.186	0.328
		20	0.998	0.999	0.818	0.884	0.827	0.881	0.635	0.734
		30	1.000	1.000	0.957	0.976	0.944	0.966	0.844	0.893
		40	1.000	1.000	0.991	0.997	0.986	0.993	0.935	0.962
		50	1.000	1.000	0.999	1.000	0.996	0.998	0.979	0.988

Table 5 The real data in Application 2.

Age (year)	Mean	StDev	Min.	Median	Max.	$\hat{\lambda}$	$\hat{\alpha}$
<1	851.2	407.2	100	824.6	1521.5	195	4
1–5	159.43	52.77	50	162.03	255	17	9
6–15	172.16	57.79	49.5	168.7	276.6	19	9
16–20	967.5	629.4	86.2	860.8	2319.1	409	2
21–30	282.4	153.6	100	262.5	599.5	84	3
31–40	348.9	203.2	100	295.4	821.1	118	3
41–50	348.9	203.2	100	295.4	821.1	118	3
>50	319.1	145	100	292.5	601.2	66	5

Age	α	T_1(*calculated*)	Decision	T_1(*calculated*)	Decision
<1	4	0.9908	A	0.9852	A
1–5	5	0.8136	R	0.7902	R
6–15	5	0.7594	R	0.7259	R
16–20	2	0.7917	R	0.7716	R
21–30	3	0.7924	R	0.7604	R
31–40	3	0.8028	R	0.7667	R
41–50	3	0.8129	R	0.7741	R
>50	5	0.8162	R	0.7753	R

A: Accept and R: Reject.

Table 6

Distribution	Test statistic T_i	Decision
Gamma$(0,1,3)$	$T_1 = 0.99633$	A
LN$(0,1)$	$T_1 = 0.95277$	R
Gamma$(1,5,3)$	$T_2 = 0.99339$	A
LN$(0,1)$	$T_2 = 0.704667$	R

A: Accept and R: Reject.

4. Sample from *LN*$(1,5)$: two-parameter lognormal distribution with μ is equal to 1 and scale σ is equal to 5.

The above four order statistics samples are used with the analogous moments of order statistics from *Gamma*$(0,1,\alpha)$, Tables 1 and 2 to run the test. The results of the tests at 5% significance level are shown in Table 6.

5. Applications

5.1. Application 1

The following data are given in Lowless (2003). The data represents the survival times in weeks for 20 males rats that were exposed to a high level radiation. The data are due to Furth, Upton and Kimball (1959) and have been discussed by Engelhardt and Bain (1977) and others. The order statistics of the data are: 40, 62, 69, 77, 83, 88, 94, 101, 109, 115, 123, 125, 128, 136, 137, 152, 152, 153, 160, 165.

By using the above data and the moments of order statistics of *Gamma*$(1,1,5)$, we calculate T_1(*calculated*) = 0.98690, T_2(*calculated*) = 0.97950. Hence from Tables 1 and 2, we recommend the gamma distribution for the given data at 5% level of significance.

5.2. Application 2

In this application, we use some collected data from Dalla hospital, Riyadh, Saudi Arabia. The data represents the cost (in SR) of 50 patients from each different ages they already have visited the outpatients clinic during one year. The summary of the data is given in Table 5. The values of $\hat{\lambda}$ and $\hat{\alpha}$ in Table 5 are estimated by using the mean and variance of the original data.

By using the original data and the moments of order statistics of *Gamma*$(0,1,5)$, we calculate T_1(*calculated*) and T_2(*calculated*). Next, we use the corresponding values and at 1% level of significance. We have the decisions in Table 6. From Table 6, we recommend gamma distribution for the age less than one year.

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التمييز بين توزيعي جاما واللوغاريتم الطبيعي مع التطبيقات

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ملخص البحث. في هذا البحث سوف نستخدم في هذا البحث معامل الالتواء للمقارنة بين توزيعي جاما واللوغاريتم الطبيعي . أيضا نبين محدودية هذه الفكرة ، وبعد ذلك نستخدم عزوم الإحصاءات المرتبة من توزيع جاما في اختبار معامل الارتباط. بالإضافة الى حساب قوة الاختبار المبنية على عدة توزيعات بديلة منها التوزيع الطبيعي، اللوغاريتم الطبيعي، ووايبل، و كوشي، خليط من التوزيع الاسي . في النهاية نقدم دراسة محاكاة وتطبيقات لبيانات فعلية.